Introduction to the Comprehensive Measurement and Management of Market Risk

XXXIII Heidelberg Physics Graduate Days

Heidelberg, October 9th, 2014

Agenda

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Overview and Definitions

What is Market Risk?

Financial Instruments - Definitions

There are **various definitions of financial instruments**, e.g.:

» **§1 (11) KWG**:

Finanzinstrumente im Sinne der Absätze 1 bis 3 und 17 sowie im Sinne des § 2 Abs.1 und 6 sind abweichend von § 1a Abs. 3 Wertpapiere, Geldmarktinstrumente, Devisen oder Rechnungseinheiten sowie Derivate. Wertpapiere sind, auch wenn keine Urkunden über sie ausgestellt sind, alle Gattungen von übertragbaren Wertpapieren mit Ausnahme von Zahlungsinstrumenten, die ihrer Art nach auf den Kapitalmärkten handelbar sind, insbesondere

1. Aktien und andere Anteile an in- oder ausländischen juristischen Personen, Personengesellschaften und sonstigen Unternehmen, soweit sie Aktien vergleichbar sind, sowie Zertifikate, die Aktien vertreten,
2. Schuldtitel, insbesondere Genussscheine, Inhaberschuldverschreibungen, Orderschuldverschreibungen und Zertifikate, die diese Schuld vertreten,
3. sonstige Wertpapiere, die zum Erwerb oder zur Veräußerung von Wertpapieren nach den Nummern 1 und 2 berechtigen oder zu ein ng führen, die in Abhängigkeit von Wertpapieren, von Währungen, Zinssätzen oder anderen Erträgen, von Waren, Indices oder Messgrößen bestimmt wird,
4. Anteile an Investmentvermögen, die von einer Kapitalanlagegesellschaft oder einer ausländischen Investmentgesellsch

Geldmarktinstrumente sind alle Gattungen von Forderungen, die nicht unter Satz 1 fallen und die üblicherweise auf dem n, mit Ausnahme von Zahlungsinstrumenten. Derivate sind

1. als Kauf, Tausch oder anderweitig ausgestaltete Festgeschäfte oder Optionsgeschäfte, die zeitlich verzö n Wert sich unmittelbar oder mittelbar vom Preis oder Maß eines Basiswertes ableitet (Termingeschäfte) mit Bezug auf die folgenden Basiswert
   a) Wertpapiere oder Geldmarktinstrumente,
   b) Devisen oder Rechnungseinheiten,
   c) Zinssätze oder andere Erträge,
   d) Indices der Basiswerte des Buchstaben a, b oder c, andere Finanzindices
   e) Derivate;
2. Termingeschäfte mit Bezug auf Waren, Frachtsätze, Emissionsbere re physikalische Variablen, Inflationsraten oder andere volkswirtschaftliche Variablen oder sonstige Vermögenswerte, Indices oder Messw
   a) durch Barausgleich zu erfüllen sind oder einer Vertra nen Barausgleich zu verlangen, ohne dass dieses Recht durch Ausfall oder ein anderes Beendigungsereignis begründet ist,
   b) auf einem organisierten Markt oder in eine stem geschlossen werden oder
   c) nach Maßgabe des Artikels 38 Abs. 1 /2006 der Kommission vom 10. August 2006 zur Durchführung der Richtlinie 2004/39/EG des Europäischen Parlaments und des Rates betre nten für Wertpapierfirmen, die Meldung von Geschäften, die Markttransparenz, die Zulassung von Finanzinstrumenten zum Handel und Sinne dieser Richtlinie (ABl. EU Nr. L 241 S. 1) Merkmale anderer Derivate aufweisen und nicht kommerziellen Zwecken dienen und nicht die Voraussetzu s 38 Abs. 4 dieser Verordnung gegeben sind, und sofern sie keine Kassageschäfte im Sinne des Artikels 38 Abs. 2 der Verordnung (EG) Nr. 1287/2006 sin
3. finanzielle Differenzgeschäfte;
4. als Kauf, Tausch oder anderweitig ausgestaltete Festgeschäfte oder Optionsgeschäfte, die zeitlich verzögert zu erfüllen sind und dem Transfer von Kreditrisiken dienen (Kreditderivate);
5. Termingeschäfte mit Bezug auf die in Artikel 39 der Verordnung (EG) Nr. 1287/2006 genannten Basiswerte, sofern sie die Bedingungen der Nummer 2 erfüllen.

Very detailed legal definition …

» **IAS 39.8**: A financial instrument is any contract that gives rise to a financial asset of one entity and a financial liability or equity instrument of another entity.

› **Here: All contracts agreed upon and all assets traded in the financial market**

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Financial Instruments - Examples

There is **no unique classification** of financial instruments. Every classification depends on its purpose and may therefore refer to different aspects of the financial instruments (e.g. accounting categories, financial mathematics aspects).

Here is an example of very simple **classification** according to the **main market risk drivers**:

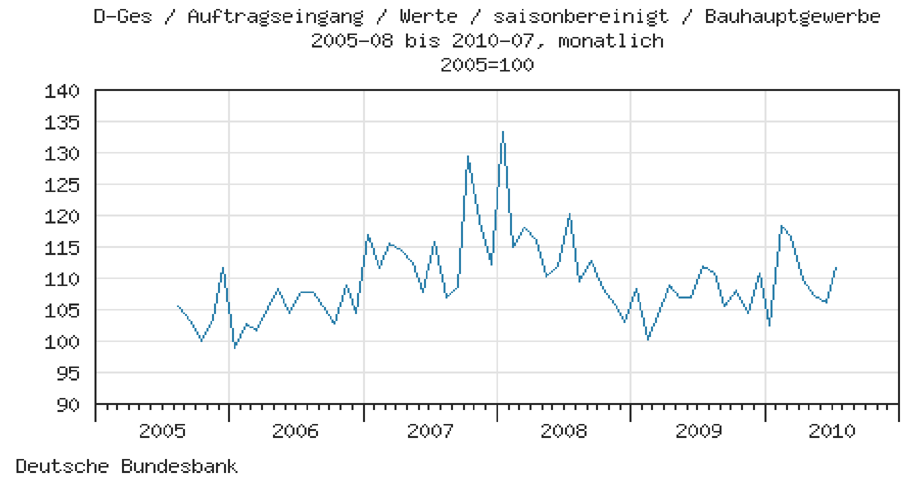
Interest rate products	Equity products	FX products	Commodity Futures/ Forwards	Credit products
• Corporate Bonds • Government Bonds • Swaps • Loans • Bond Options • Swaptions • Caps/Floors • IR Futures • …	• Shares • Equity Options • Index Options • Equity Certificates • Basket Options • ...	• FX Cash • FX Forwards • FX Options	• Crude Oil • Gas Oil • Base Metals • Precious Metals • Coal • Power • …	• Credit Default Swaps • Securitisations • Credit Index Products • Credit Basket Derivatives • ...

Different financial instruments are sensitive to different aspects of market risk.

Market Risk of Financial Instruments – Motivation

The **current (market) value** of a financial instrument is subject to the current **market conditions**. Therefore the value of any financial product will usually **fluctuate** from one day to another (and also during the day) due to changes in the market conditions.

Market conditions can be quantified by (or from) quantities that can be observed in the market, e.g., exchange rates or equity courses.



The quantities that quantify market conditions reflect the **market's current opinion**, which in turn depend on more or less **unpredictable economic variables** (e.g. unemployment rate, company news, GDPs, …) and **psychology** (the reactions of the market to any news are also more or less unpredictable).

Therefore the quantities quantifying the market conditions need to be modeled as **stochastic variables**.

Market Risk Definition: Potential losses in the value of financial instruments due to (stochastic) changes in the market conditions.

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Classification of Market Risk

Quantification of Market Conditions – Risk Factors

The **current (market) value** of a financial instrument is subject to **market risk** due to the stochastic nature of the market conditions. On the other hand, once the market conditions are specified / quantified the corresponding current value of financial instruments is fixed. The **pricing functions are deterministic** functions that relate the current market conditions to the current price/value.

Example: Portfolio of 10 Intel Corp. stocks (Sep 27th)

- Price: 19.42 USD
- EUR/USD exchange rate: 1 EUR = 1.3442 USD
- Portfolio-Value:

  10 · 19.42 USD / 1.3442 (USD/EUR) = <u>144.47 EUR</u>

- For the example portfolio above the relevant **market conditions** are specified by the Intel stock price and the USD/EUR exchange rate.
- Stock price and exchange rate are **valuation parameters** for the pricing function.
- Stock price and exchange rate are also the **sources of market risk** for the portfolio and are therefore called **risk factors**.

Risk factors quantify market conditions and are the sources of market risk.

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Quantification of Market Conditions – Risk Factors

For **quantitative measurements** of market risk the market conditions need to be formulated in **mathematically tangible quantities** and structures that can be represented by numbers, vectors or arrays. These quantities and structures or their elements are called **risk factors**.

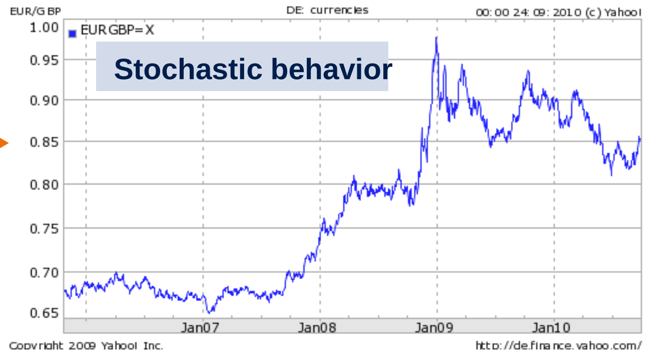
Examples:

» Equity prices: BASF, E.on, Toyota, Pfizer, …

» Exchange rates: EUR/XAU, EUR/GBP, …

» Equity Indices: Dax, MDax, Nikkei 225, …

» Commodity spot prices: Brent, WTI, XAG, …



» **Risk factors** are closely related to the **valuation parameters** of pricing functions (see previous slide).

» Current information about the risk factors can be **extracted from the market** and also historical data is usually available (can be bought).

» From these data **time series** can be constructed that allow the analysis of the risk factor's **stochastic properties**.

Stochastic properties of risk factors are determined from historical time series.

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Risk Factors – Return Types

Usually it is not the risk factors themselves but their **returns** that are **used for market risk calculations** (therefore sometimes the returns are called risk factors). This is because the returns can **usually be better described by basic stochastic processes** than the risk factors themselves and in risk management one is more interested in the changes of quantities than in their absolute values.

A risk factor return r describes the change in a risk factor ρ between two points in time. There are three kinds of returns used for the different risk factors:

» absolute return: $$r_{abs} = \rho_t - \rho_{t_0}$$

» relative return: $$r_{rel} = \left(\rho_t - \rho_{t_0}\right) / \rho_{t_0}$$

» logarithmic return: $$r_{\log} = \ln\left(\rho_t \middle/ \rho_{t_0} \right)$$

The **choice** of the return type depends on **characteristics** of the respective risk factor.

Market risk is measured via absolute, relative or logarithmic risk factor returns.

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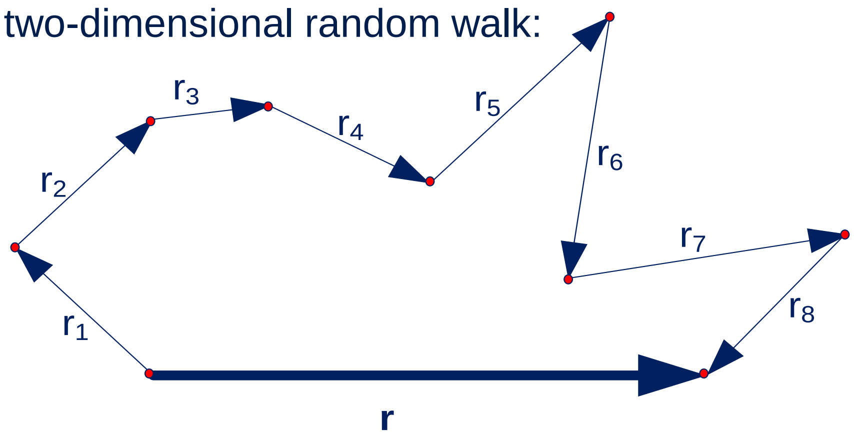
Risk Factors – Assumption of Stochastic Processes

For risk management calculations one has to **assume stochastic process models** for the risk factor returns or make **direct distribution assumptions** for them.

» **Market risk measurement is based on phenomenology.**

A widely used assumption especially in the equity market is the assumption that logarithmic returns of equity prices follow a one-dimensional **random walk**: each incremental price movement is **independent** and **identically distributed**.

End-to-end vector of two-dimensional random walk:



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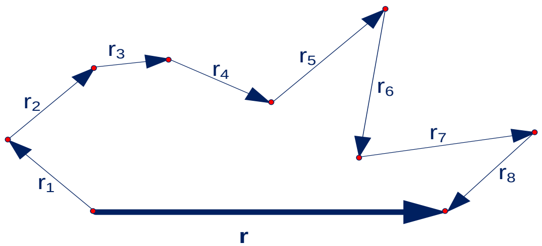
Risk Factors – Stochastic Processes

Random Walk



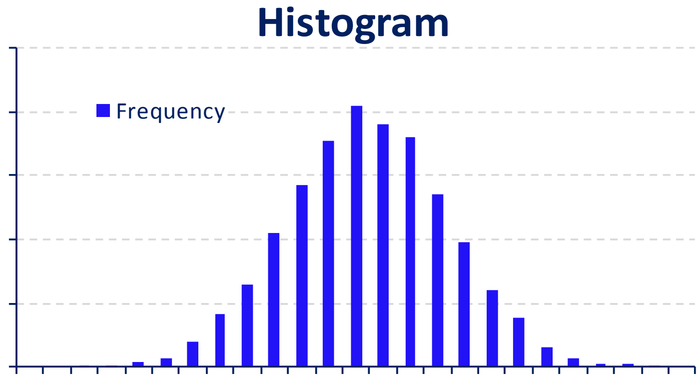
For the number of steps n (n=8 in figure) sufficiently large the distribution of rlog (end-to-end vector r in figure) follows a **normal distribution**:

$$p(r_{\log}) = \frac{1}{\sqrt{2\pi \operatorname{var}(r_{\log})}} \exp\left[\frac{\left(r_{\log} - E[r_{\log}]\right)^2}{2\operatorname{var}(r_{\log})} \right]$$

$$E[r_{\log}] = 0 \quad , \quad \operatorname{var}(r_{\log}) = n \cdot E[r_i^2]$$

The **number of steps** in the random walk n is proportional to the **time Δt** that passes during the random walk.

This also holds for **absolute returns** (relative returns do not form a random walk but one can still assume that they are normally distributed).



Random walk is used as process model for risk factors.

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Risk Factors – Stochastic Processes

Replacing *n* by the time for the random walk (t-t_0) and the constant of proportionality by σ^2 the expectation value and the variance can be written as:

$$E\,[r_{\log}] = 0 \qquad , \qquad \mathrm{var}(r_{\log}) = \sigma^2 (t - t_0)$$

Due to the self-similarity of the random walk these relations also hold in the limit of infinitesimal time intervals:

$$E\,[dr_{\log}] = 0 \qquad , \qquad \mathrm{var}(dr_{\log}) = \sigma^2 dt$$

This is very similar to the well known **Wiener process**, whose random movements dW are normally distributed with expectation value 0 and variance equal to the time of process (dt):

$$dW \sim X \sqrt{dt} \qquad \text{with} \qquad X \sim N(0,1)$$

By comparison with the Wiener process (var(dW) = dt) we can write the stochastic process for the log-returns as:

$$dr_{\log} = \sigma \cdot dW$$

Stochastic processes for log-returns are related to Wiener processes.

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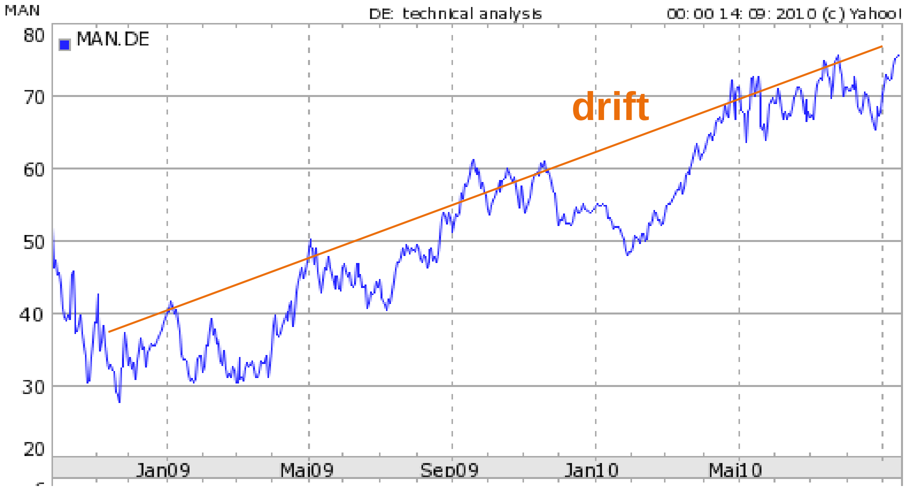
Risk Factors – Stochastic Processes

So far the process model has only one parameter σ, which is called volatility and can be written as:

$$\sigma = \sqrt{\frac{1}{(t-t_0)}\operatorname{var}(r_{\log})}$$

while the expectation value of the stochastic process for the log-return is always 0.



Risk factors (if they are tradable) are **martingales** which means that their current value is given by their discounted future expectation value. To reflect this in the process model we add a deterministic drift term:

$$dr_{\log} = \mu \cdot dt + \sigma \cdot dW$$

Because the **drift** term is deterministic dr_{log} is still normally distributed with variance $\sigma^2(t-t_0)$ but now the expectation value is $\mu(t-t_0)$.

This stochastic process is usually used to model equity prices implying log-normally distributed equity prices.

Stochastic processes for equity prices are often modeled as diffusion process.

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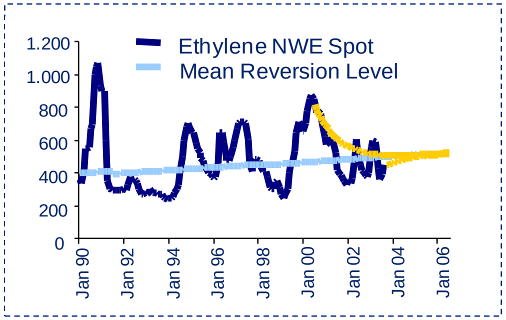
Risk Factors – Stochastic Processes

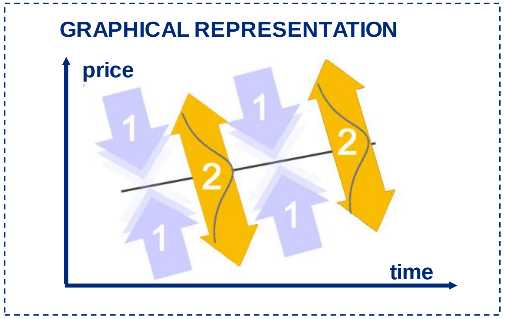
The simple process model can be generalized to a **general diffusion process** with drift and variance being functions of t and ρ:

$$d\ln(\rho_t) = a(t,\rho_t)\cdot dt + b(t,\rho_t)\cdot dW$$

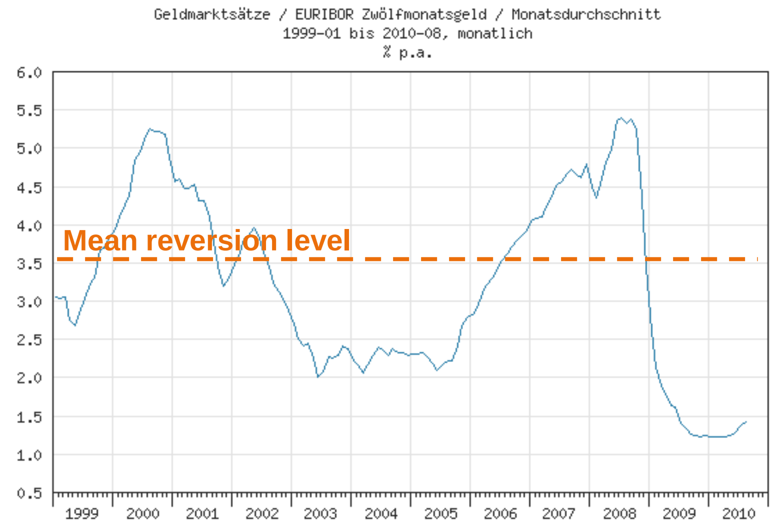
(**lognormal model**)

In this way the model can also be used for risk factors that show different (e.g. mean reverting) behavior like commodity prices or FX rates.





Example **Vasicek model**:
(Ornstein-Uhlenbeck process)



$$d\rho_t = (\mu - \nu\rho_t)\cdot dt + \sigma\cdot dW$$

(**normal model**)

Mean reversion force

The simple diffusion model can be generalized to more complex processes.

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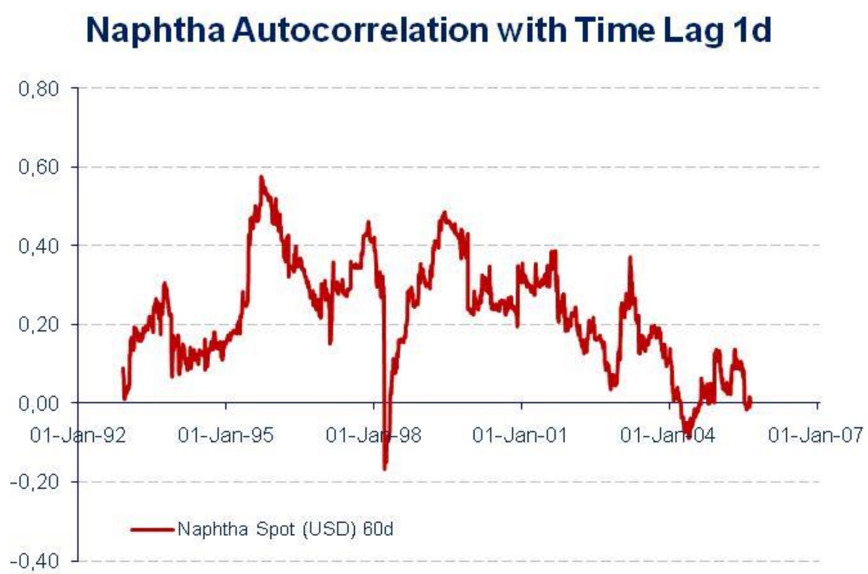
Risk Factors – Stochastic Processes

Assumption in random walk model:

Assuming that the **current value** of a risk factor, such as a stock price or an interest rate, **contains all the information** about its historical development (this is called weak market efficiency), it follows that the subsequent values taken on by such a risk factor **depend** only on the current price and other external effects, such as politics, but **not on the past prices** or rates. Market prices can then assumed to be **Markov processes**.



This assumption is not always valid leading to generalizations of the random walk model to **autoregressive process models** (e.g. AR, GARCH models).

Example AR(1) model:

$$X_t = \phi X_{t-1} + \varepsilon_t \qquad \text{with} \qquad \varepsilon_t \sim N(0,\sigma^2)$$

Risk factors are usually modeled as Markov processes.

Risk Factors – Model Simplification for Market Risk Calculations

Most **market risk calculations** in banks are performed on **short time horizons** of only a few days. As we will see later this is also connected to regulatory requirements. For these purposes the **restriction** of modelling risk factors as simple processes containing only a constant drift and a **stationary diffusion** term is usually sufficient:

- » lognormal model $$d\ln(\rho_t) = \mu \cdot dt + \sigma \cdot dW$$

drift **volatility**

- » normal model $$d\rho_t = \mu \cdot dt + \sigma \cdot dW$$

In the following we will therefore concentrate on these processes. However, for **pricing purposes** (e.g. for interest rate models) or **for more sophisticated risk calculations** (e.g. on longer time horizons) **more complex processes** are required.

The next step is to calibrate the model by determining the process parameters drift and volatility from available (historical) market data.

Market risk: risk factors are usually modelled as simple diffusion processes.

Risk Factors – Determining Process Parameters from Time Series

Process Parameters

Based on the assumption of **stationary markets** drift and volatility of the stochastic process can be calculated from time series of the risk factor returns.

$$\mu = E[r_{t/t_0}] \quad , \quad \sigma = \sqrt{\frac{1}{(t-t_0)} \mathrm{var}(r_{t/t_0})}$$

This volatility is the **historical volatility** of the respective risk factor. Besides the historical volatility there is also the **implied volatility**. In contrast to the historical volatility that is calculated from historical data the implied volatility is derived from current market data (e.g. option prices, compare lecture on valuation of equity derivatives).

Practical Difficulties – Data Quality

- **Missing data**
  - small gaps: constant/linear interpolation; proxy time series return
  - larger gaps: Brownian Bridge or EM algorithm
- **Products without available data history**
  - map on suitable existing time series (related product)
  - create synthetic time series with desired volatility and correlation

StdDev
Mean

date	Dax	r_{log}
24. Sep 10	6.298,30	0,0182
23. Sep 10	6.184,71	-0,00381
22. Sep 10	6.208,33	-0,01084
21. Sep 10	6.275,98	-0,00296
20. Sep 10	6.294,58	0,013567
17. Sep 10	6.209,76	-0,0064
16. Sep 10	6.249,65	-0,00195
15. Sep 10	6.261,87	-0,00216
14. Sep 10	6.275,41	?
13. Sep 10	?	?
10. Sep 10	6.214,77	-0,00109
09. Sep 10	6.221,52	0,009217
08. Sep 10	6.164,44	0,00758
...	...	...

Market risk calculations are based on historical data.

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Risk Factors – Risk Factor Categories

Risk factors can be divided into groups according to their market risk category:

Interest Rate	Spread	FX
Volatility	Equity	Commodity

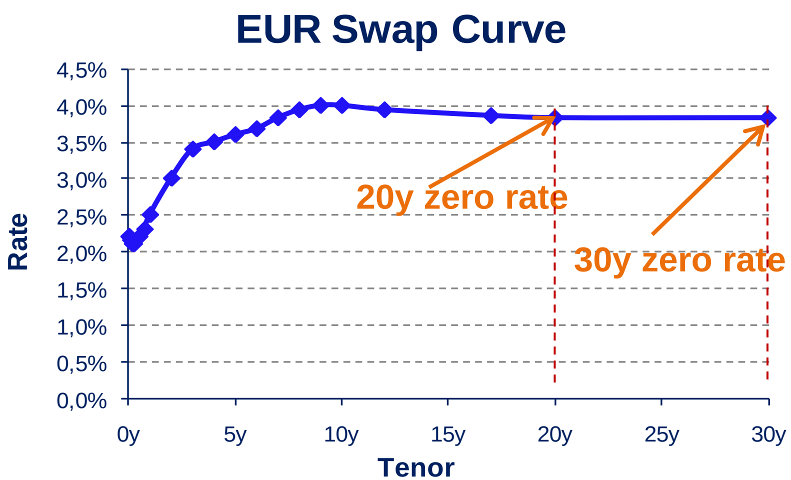
Within these categories there are still different kinds of risk factors:

- 0-dimensional structures: prices, indices, FX rates
- 1-dimensional structures: interest rate curves, spread curves, commodity forward curves
- 2- or 3-dimensional structures: equity volatility surfaces, FX volatility surfaces, interest rate volatility surfaces (cap/floor 2-dim, swaption 3-dim)

There are different categories and types of risk factors.

Risk Factors – Interest Rate Curves

The interest rate determines the interest one receives when depositing money in a bank. The rate depends on the time the money is deposited, therefore interest rates have a **term structure**. Another way to look at it is the **time value of money**. One euro today is worth more than one euro received in a year.



Specification requires:

- **Interpolation method**: e.g., linear on DF, ...
- **Day count convention**: A/360, Act/Act, ...
- **Business Day Convention**: preceding, ...
- **Compounding**: continuous, 3m, 6m, ...

There are different kinds of interest rate curves depending on the referenced entity:

- **Government curves**: Germany, France, UK, US, …
- **Swap curves**: EUR, USD, GBP, JPY, …

Depending on the properties of the financial instruments they are constructed from Government and Swap curves can be further differentiated (see next slide).

Risk factors

Interest rate curves are usually defined as array of zero rates (marker on diagram curve) for a fixed grid of tenors. The zero rates per tenor can be used as risk factors for the rate curve (generate time series for each tenor from daily observation).

Interest rate curve: simple concept, complex in practice.

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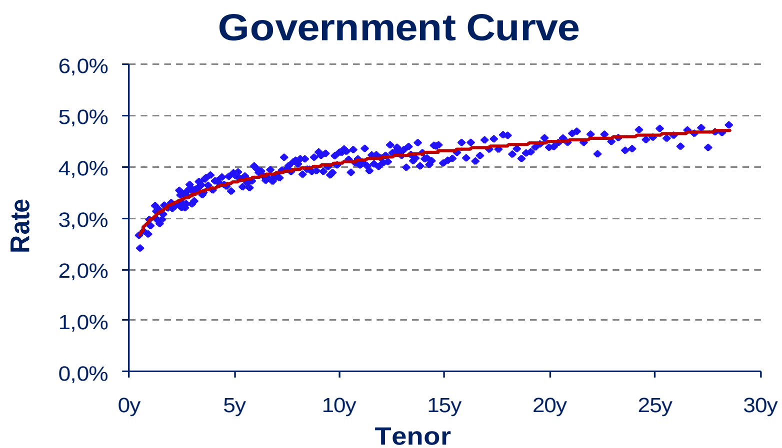
Risk Factors – Interest Rate Curves

Construction of Interest Rate Curves

Government curves

A government curve is constructed from the prices of government bonds. It can be calculated as regression curve through the available data but often also parametric forms are used.



There are also special government curves that are constructed from government bond futures (Germany: Bund, Bobl, Schatz) only.

Swap curves

During the recent financial crisis the classification of swap curves has been further differentiated. Now banks use several swap curves for the same currency depending on the instruments they are constructed from.

Swap curves are constructed via **bootstrapping** from different instruments:

- short end: deposits, FRAs, futures, EONIAs
- long end: swaps

Today the **different degree of credit risk and liquidity** of the different instruments can no longer be neglected in the construction of rate curves. The same holds for basis risks between swap tenors.

Interest rate curve construction has become more complex during the financial crisis.

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Risk Factors – Spread Curves

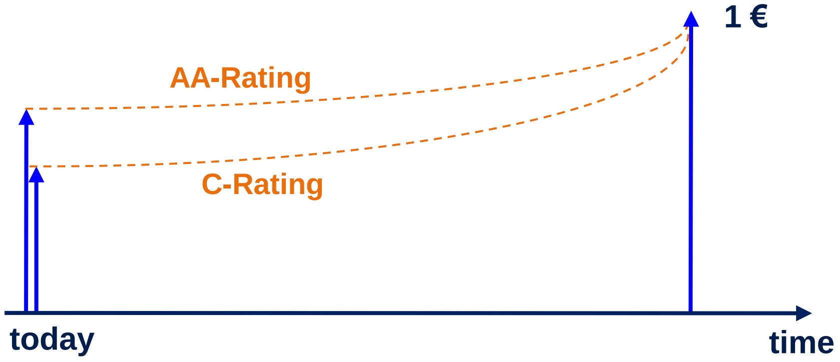
Creditworthiness and Spread Curves

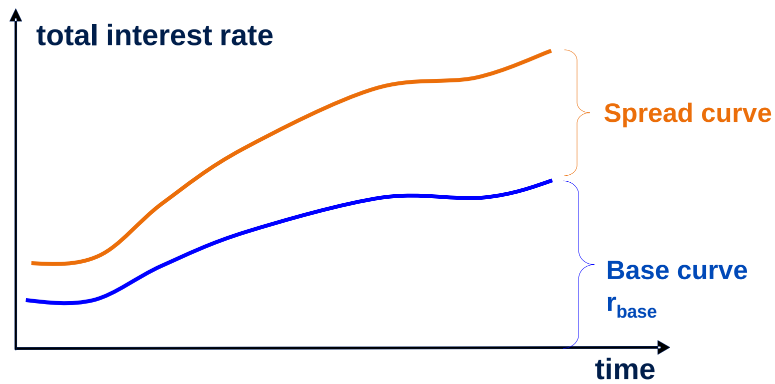
The value of financial instruments and contracts depends on the **creditworthiness** of the issuer or counterparty. This is reflected in the time value of money. 1 euro due in one year from a AA-rated company is worth more than 1 euro due in one year from a C-rated company.

To reflect the creditworthiness of issuers and counterparties in valuation and risk measurement of financial instruments **individual rate curves** are necessary. On the other hand risk management usually wants to **separate risk contributions** of interest and creditworthiness.

Therefore one introduces so called **spread curves** that measure the spread between a common base curve (e.g. swap curve) and the issuer/counterparty specific curve.

Market risk accounts only for the **change in spread levels** but **not for defaults or rating migrations**!



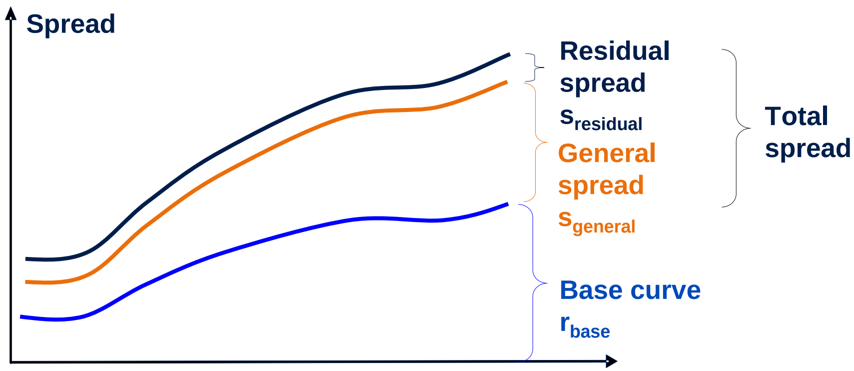


Spread curves reflect the creditworthiness of the issuer/counterparty.

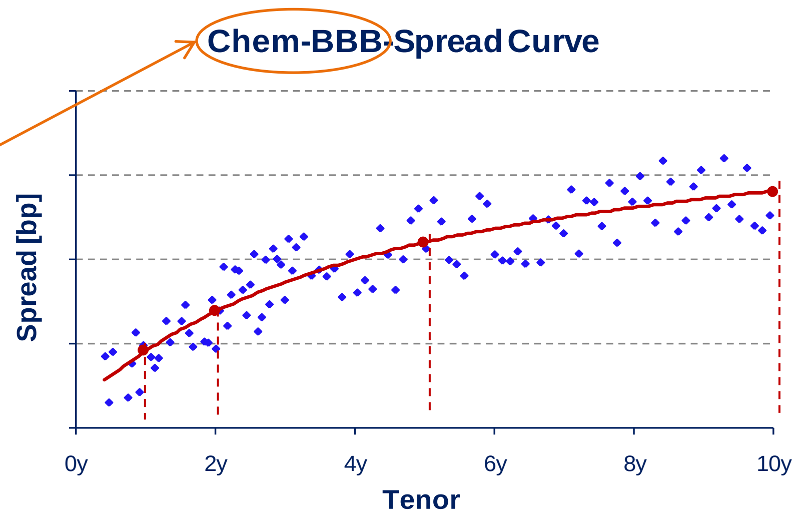
Risk Factors – Spread Curves

Spread Curve Construction (Bonds)

In **practice** one often **lacks** the **data** to generate individual spread curves for each address. Instead general spread curves (per sector/rating) with a residual spread are used.



$$r_{total} = \underbrace{r_{base} + s_{general}}_{\text{systematic}} + \underbrace{s_{residual}}_{\text{idiosyncratic}}$$



Spread Curve Risk Factors

» In analogy to the case of interest rate curves spreads for individual tenors are used as risk factors for the general (sector/rating) spread curves.

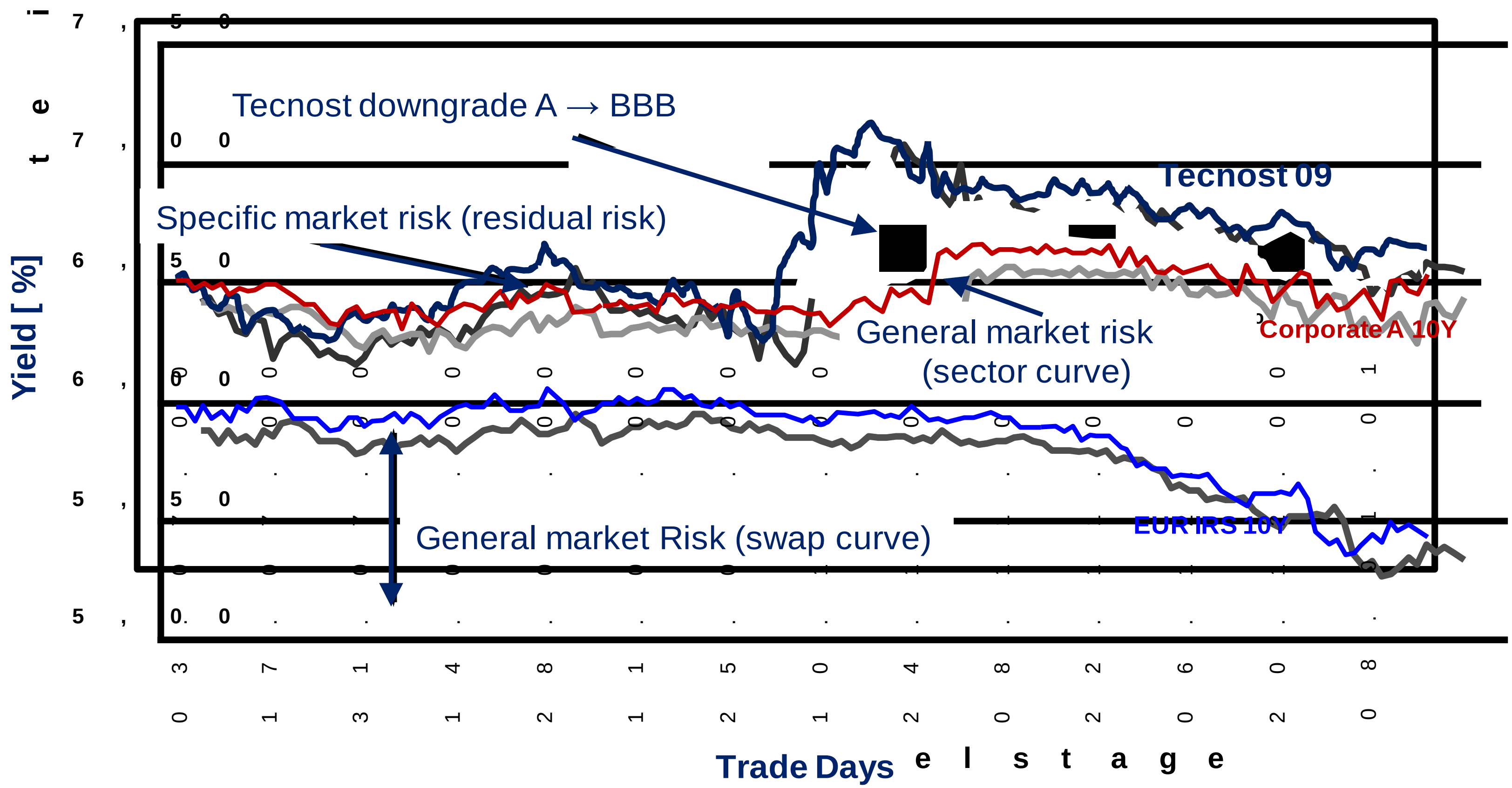
» The residual spreads are usually modelled as single risk factor.

» For CDS often issuer specific spread curves are used.

Spread curves can be separated into systematic and idiosyncratic parts.

Risk Factors – Spread Curves

Example of Spread Curve Risk



Market risk (spreads) does not account for defaults or rating migrations.

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Risk Factors – Volatility Surfaces

Volatility Surfaces (implied volatilities)

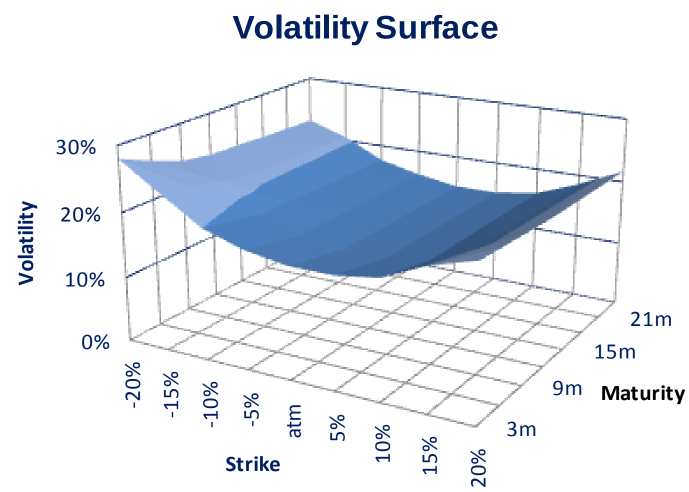
The volatility that can be extracted from the time series of risk factors is called historical volatility. The **historical volatility** is used for market risk calculations.

For pricing volatility dependent instruments the historical volatility is not adequate, instead the **implied volatility** (the volatility that is implied by the price) has to be used. The implied volatility is calculated from option prices by solving the pricing function for the volatility.

Volatility surfaces are 2-dimensional objects (swaption volatility surfaces even have 3 dimensions) that define volatility as function of time to maturity and strike of the option.

Examples:

- » **IR:** caplet-, swaption-, bond option-surfaces
- » **EQ:** stocks, indices
- » **FX:** major currency pairs



Volatility Surface Risk Factors

The risk factors are given by fixed grid points on the maturity-strike grid. The time series for the risk factors are generated from daily observations of the volatility surface.

Implied volatilities are used as risk factors.

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Risk Factors – Portfolio Level

Market Risk Factors on Portfolio Level

- The trading book of big banks contains all kinds of risk factors that easily amount to **several thousand risk factors**. Risk measurement has to take all (relevant) risk factors into account.
- Risk factors usually are not independent of each other but are **correlated**. Risk measurements must take these correlations into account.
- Risk factors continuously vary while markets are open. Therefore risk factor **observations** to generate time series for risk measurement **must be simultaneous** to preserve correlations.
- Portfolios are often hedged to eliminate risks. This might result in risks that can be neglected on instrument level to be dominant on portfolio level.

Adequate risk measures, tools and processes are needed to manage market risk.

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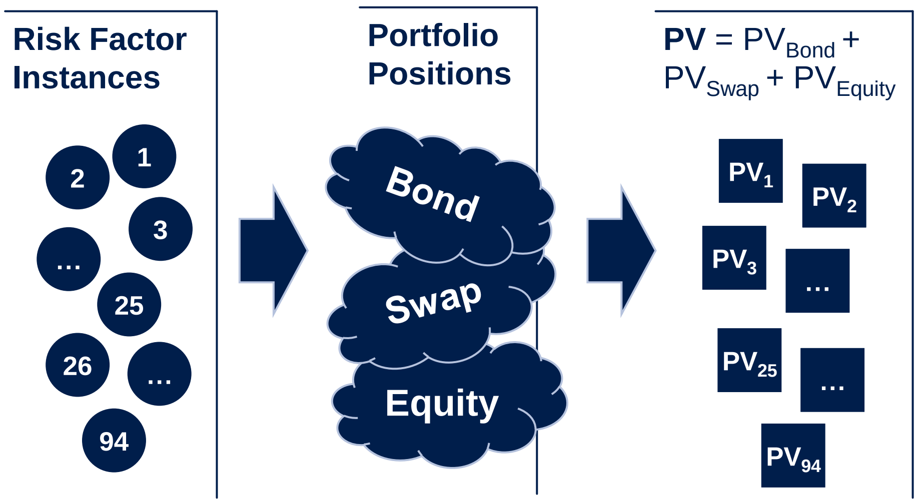
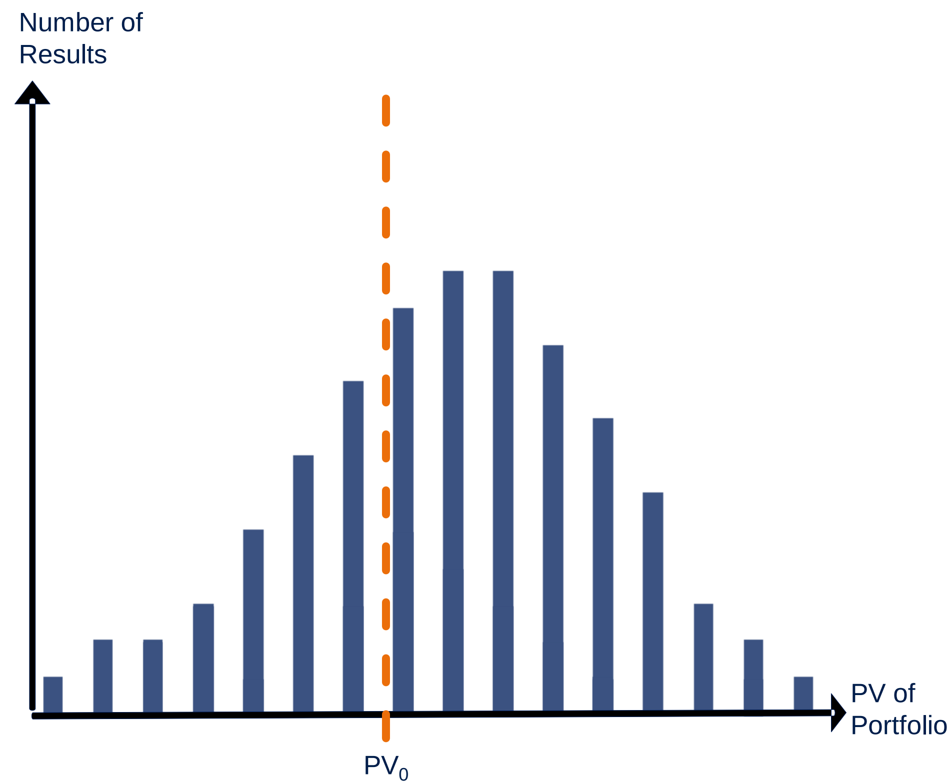
Market Risk Valuation – Portfolio Level

Random Change of Risk Factors

Going forward in time risk factors change. The Profit-and-Loss (P&L) is derived as the difference between the new (PV_i) and old present values (PV_0), first for single portfolio positions and subsequently for the portfolio.





The Profit-and-Loss distribution derives from the (common) distributions of risk factor changes.

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Measurement of Market Risk

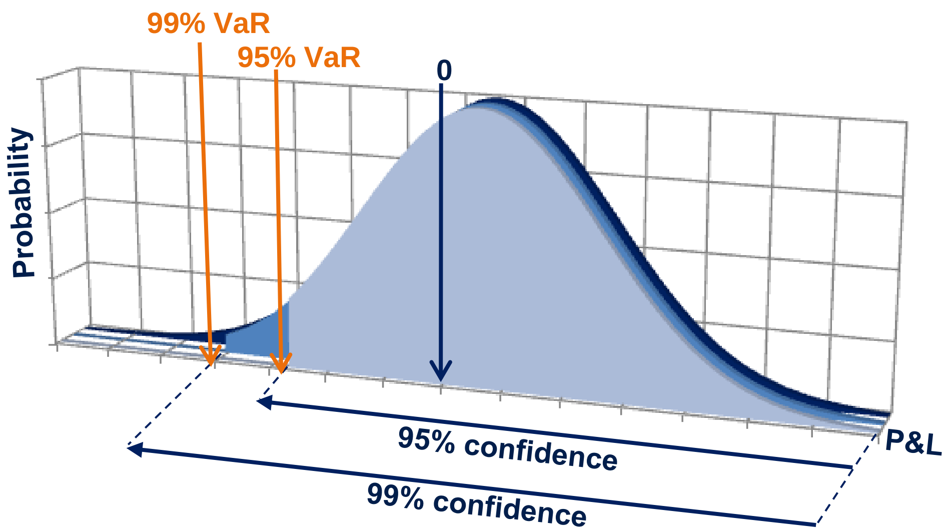
Introduction of Market Risk Measures – Value-at-Risk (VaR)

Market Risk of Portfolios

Financial instruments and therefore portfolios of financial instruments are subject to market risk.

Market risk management requires adequate measure to quantify market risk of portfolios that are sensitive to thousands of risk factors.

Estimated Portfolio P&L Distribution



Value-at-Risk (VaR)

The Value-at-Risk of a portfolio over a certain time horizon T is the maximum loss which will not be exceeded with a given probability (one sided confidence level).

» VaR is measured in monetary units (e.g. €).

» VaR can be calculated for arbitrary quantiles.

» VaR is usually calculated on short time horizons (e.g. 1 or 10 days).

» VaR is based on historical risk factor time series from which the estimated portfolio P&L distribution is calculated (assumption of stationary market).

VaR quantifies potential portfolio loss based on historical data.

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Introduction of Market Risk Measures – Expected Shortfall

Expected Shortfall (Expected Tail Loss)

The expected shortfall (ES) is also called expected tail loss or conditional VaR.

It is related to VaR and measures the expectation value of losses (as a positive number) that exceed the VaR.

» ES is measured in monetary units (e.g. €).

» ES can be calculated for arbitrary quantiles

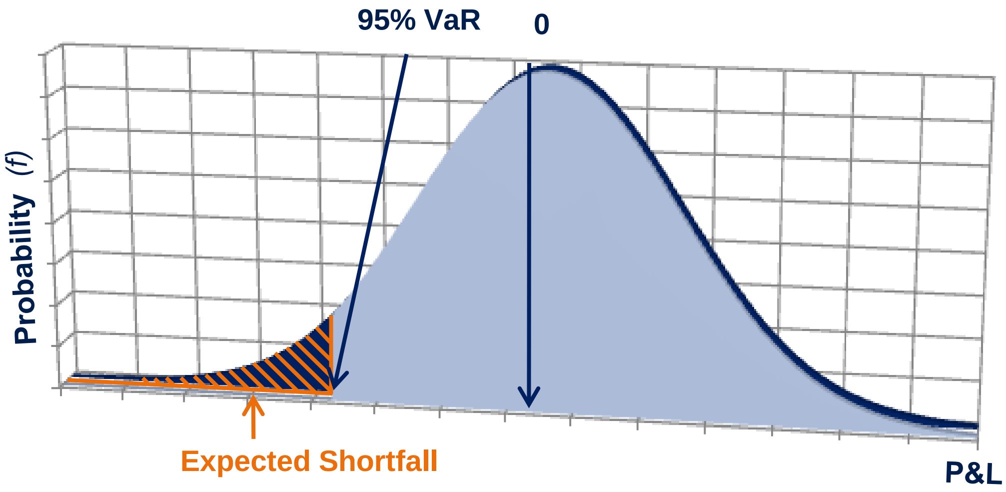
» ES is usually calculated on short time horizons (e.g. 1 or 10 days)

» ES is based on historical risk factor time series.

» In contrast to VaR ES is a coherent risk measure and therefore is sub-additive: for two sub-portfolio *A* and *B* holds:

$$ES(A+B) \leq ES(A) + ES(B)$$

Estimated Portfolio P&L Distribution



$$ES(\alpha) = \frac{\int_{-\infty}^{VaR(\alpha)} PL \cdot f(PL)dPL}{\int_{-\infty}^{VaR(\alpha)} f(PL)dPL}$$

Expected Shortfall is coherent risk measure that is related to VaR.

Why Measuring Market Risk?

Introduction to the Regulation of Financial Institutions

Legal Foundations for German Banking

German Commercial Code (HGB)
Handelsgesetzbuch (10.05.1897)

- Specific civil law of trading
- Extending German Civil Law (BGB) (subsidiarity)
- Regulates basic requirements on trading
- Special directive for financial institution
  - Accounting principles
  - Audit

German Banking Act (KWG)
Kreditwesengesetz (05.12.1934)

- Preservation of financial industry
- Protection of creditors
- Restriction on risk-related business activities
- Imposition of extended duty of disclosure
- Ensures competences of regulators
- Legal justification of
  - German Solvability Directive (SolvV)
  - German MaRisk

HGB and KWG form the basis of German banking regulation.

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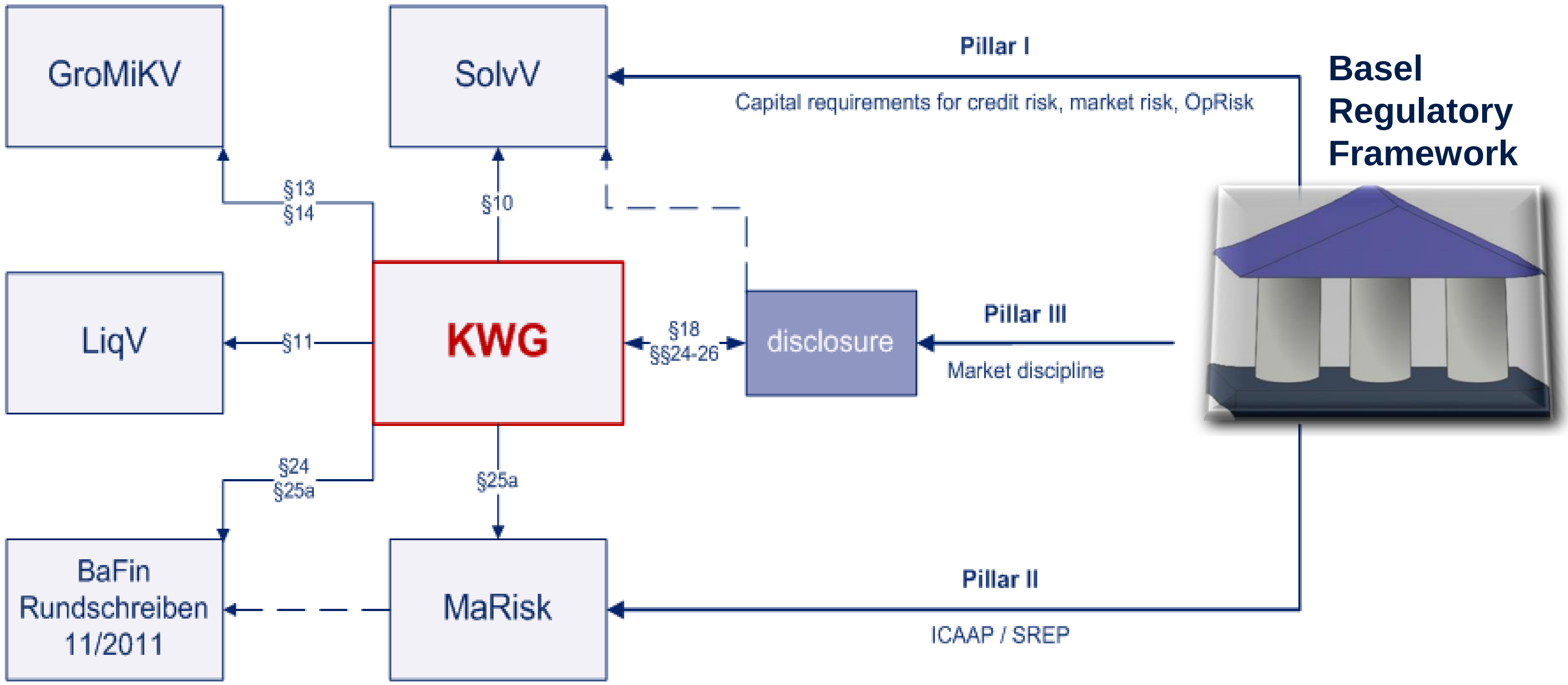
Overview of German Banking Regulation (until 2009)



German regulation enforces Basel regulatory framework.

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The Role of the Basel Committee

Basel Committee on Banking Supervision (founded 1974)

- Members: Central banks and supervisors (founded by G10, now global membership)
- Goal: Development of high consistent standards for banking and supervision
- Activity: Quarterly meetings in Basel (Bank for International Settlements)
- Results: Development of legally non-binding guidelines and recommendations through discussions with banks and regulatory authorities

Policy Development Group
Risk management/-modelling, liquidity, trading book issues, research, (supervision of) capital resources, cross-border-topics

Standards Implementation Group
Validation,
operational risk

Accounting Task Force
Conceptional issues, audit, financial standards in praxis

Basel Consultative Group
Dialogue with non-member countries on supervisory issues and initiatives of the committee

Banking regulation is strongly influenced by the Basel Committee.

Regulatory Requirements – From Basel to European & National Law

Basel Committee on Banking Supervision:
Basel III Accord

Three-Pillar-Structure

» Pillar I: Minimum (regulatory) capital requirements (solvability principles)

» Pillar II: Supervisory review process, Economic capital (SREP, ICAAP)

» Pillar III: Market disclosure and discipline (capital resources, risk profile)

Capital Requirement Regulation (CRR) (Reg. 575/2013)

Capital Requirement Directive (CRD IV) (Directive 2013/36/EU)

KWG
Solvabilitätsverordnung
MaRisk

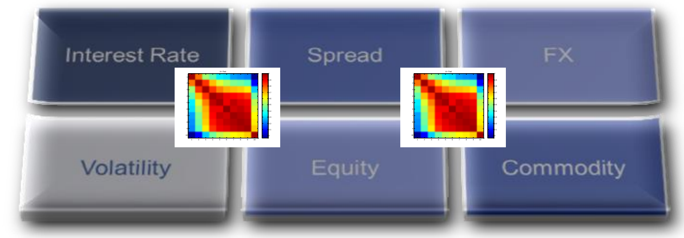
CRR is directly binding since January 2014, CRD IV has been implemented into German law.

Purpose and Goals of Banking Regulation

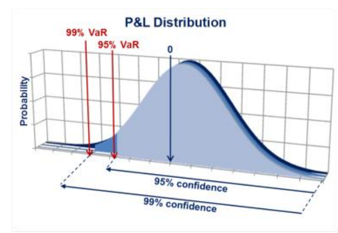
Brief Recap – (Market) Risk Modelling in a Nutshell

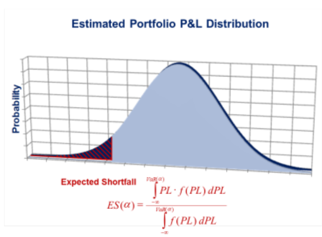
Definition of Market Risk: Potential losses in value of financial instruments due to (stochastic) changes in the market conditions.

- » Identify risk position (portfolios, desks, institute)
- » Classify and model risk factors (EQ, IR, Vol, FX)



- » Derive Profit & Loss distribution (scenario-wise)
- » Choose Risk Measure (VaR, Expected Shortfall)





- » Analyse and understand risk structure
- » Report comprehensive risk assessment

The understanding of risk is essential to grasp the necessity for regulation and recent evolutions.

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Rational Arguments on Measuring (Market) Risk – Management Impulses

Managing (Market) Risk

Financial instruments and portfolios of them are subject to (market) risk.

- How much risk?
  - Quantify risk of a portfolio and compare risk and return
    => Management impulses: Which business should be increased/reduced?
  - Quantify total risk and compare to risk capacity
    => Determine economic capital requirement
  - Limit utilisation
    => Generates direct management impulses
- Understanding business-related risk
  - Find (unknown) risk concentrations
  - Analyse risk sensitivities
  - Do hedges generate the desired risk reduction?

Business and Risk Strategies

... Seems like a good idea ...

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Goals of Regulatory Requirements

Managing Risk

Besides being a good idea, management of (market) risk is a regulatory requirement.
Regulation has two **main goals**:

» Banks shall **analyse, understand and manage risks** inherent in their business.

» Ensure minimum **capital buffers to balance these risks**.

Regulation aims at banks understanding risk and at capital requirements.

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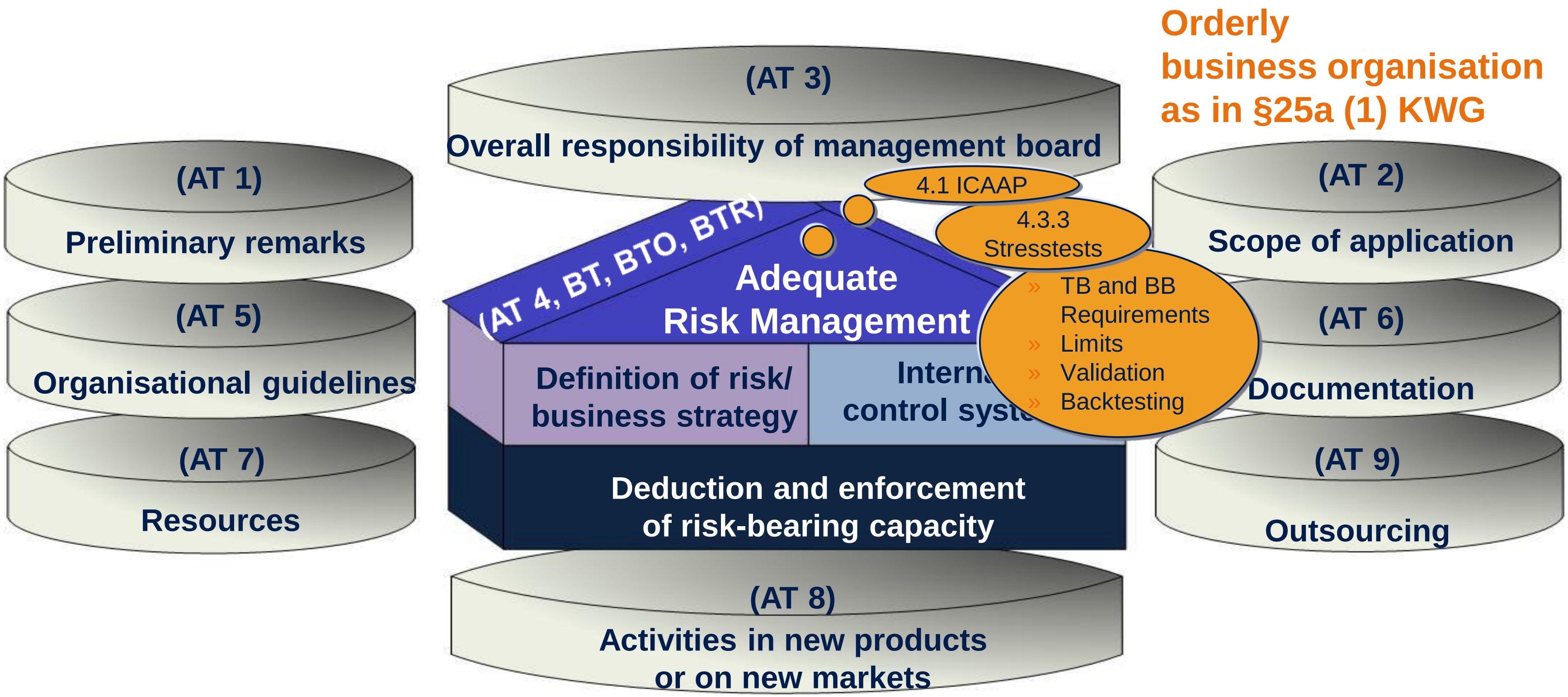
Regulatory Requirements – MaRisk

Minimum Requirements on Risk Management – MaRisk



Principles of risk management and risk-bearing capacity (→economic capital).

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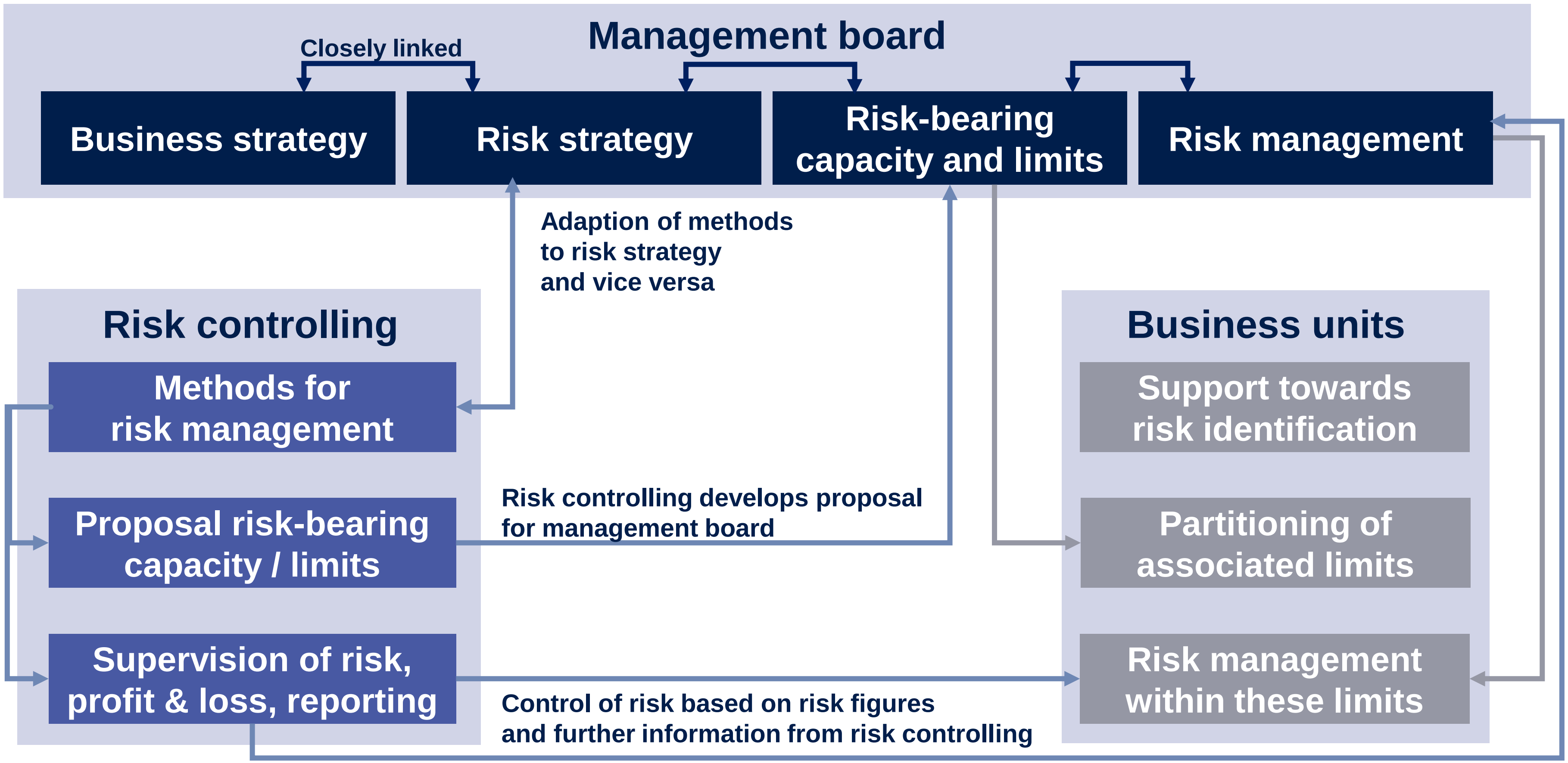
Organisation of Risk Management

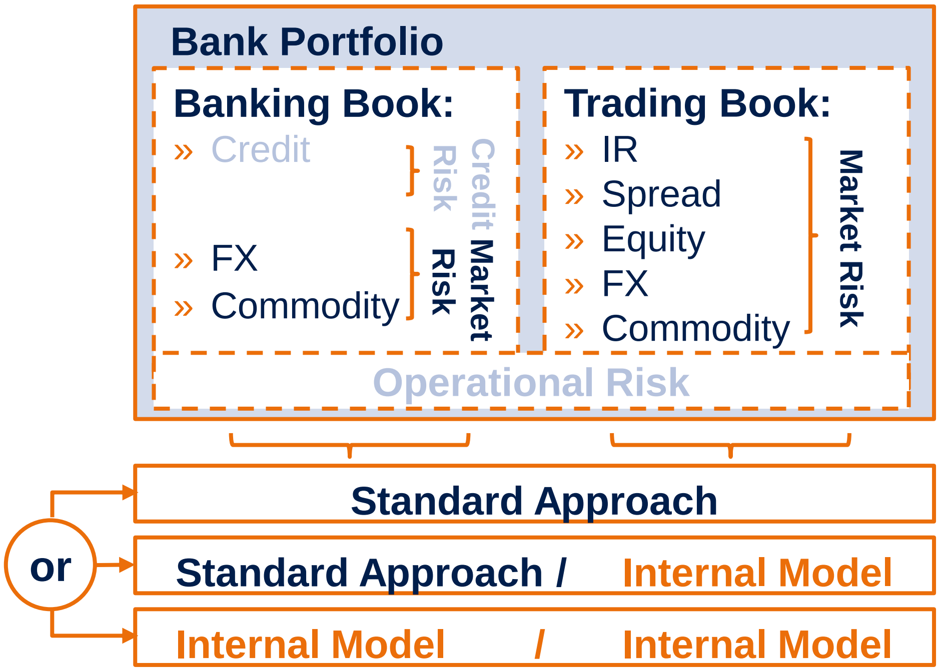


Possible real-world organisation of risk management in accordance with the MaRisk.

Requirements for Regulatory Capital

Managing Market Risk – Regulatory Capital Requirements

» Regulation requires capital buffers for positions subject to **market risk (CRR)**:



Standard Approach

In the standard approach, minimum capital requirements are basically defined by weighted exposures.

» Definition of exposure measures

» "god-given" weight factors

Internal Model Approach

Internal models are usually based on a VaR-model. Constraints for the model:

» one-sided confidence interval of 99%; 10 day holding period

» Effective historical observation period ≥1y

» Extended requirements for specific interest rate risk (since after the financial crisis)

Partial Use

Combination of both with internal model used only for selected market risk categories.

Banks must use the standard approach or an internal model.

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The Novel Market Risk Modelling Framework

New since Basel II

Internal Market Risk Models

Value-at-Risk, 10 day risk horizon, risk factor time series 1y+, 99% confidence level, factor 3+x+y

Stressed Value-at-Risk, as above, but parameterised in stress period (e. g. 2007/08), factor 3+z

General Market Risk

EQ, EQ vol

FX, FX vol

IR(1), IR vol

Specific Market Risk

Residual Risk

Issuer spreads, equities

Event Risk

Equities (M&A, jumps, etc.)

+

Exceptions Outside

Incremental Risks

1 year risk horizon + ≤1 year liquidity horizon(2), 99,9% confidence level, factor 1, default and migration risk (at least specific IR)

+

Fallback Mod. Stand. Approach

Comprehensive Risk

for the „Correlation Trading Portfolio“ (CDO tranches, n-th-to-default credit derivatives with sufficient liquidity), all relevant price risks, factor 1, floor w.r.t. modified standardised approach

(1) IR = base curve + sector-by-rating spreads; (2) Assumption: constant level of risk

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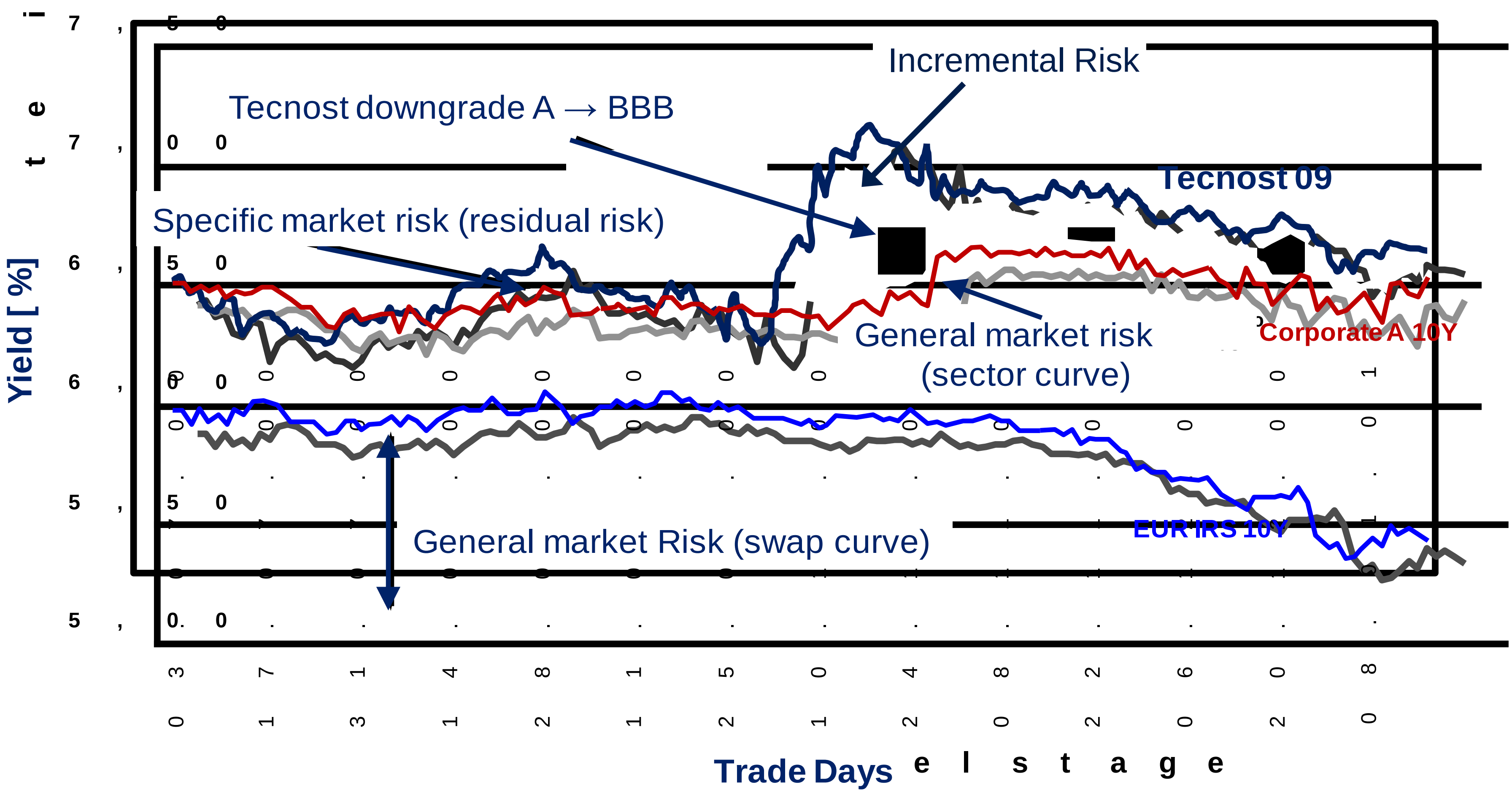
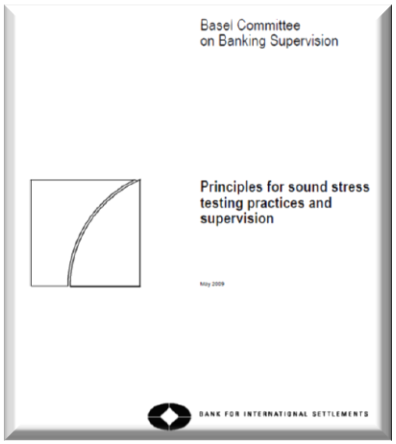
Regulatory Requirements – Risk Classification of a Bond (Example)



A New Era of Regulation

Crises and the Need for Change

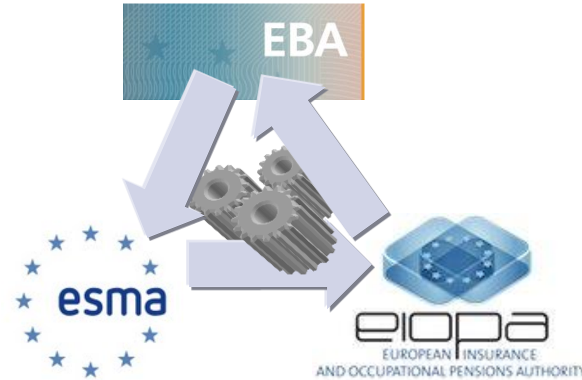


Selected Publications (non-binding standards / consultative papers)

- Principles for sound stress testing practices and supervision (05/2009)
- Enhancements to the Basel II Framework (07/2009)
- Revisions to the Basel II market risk framework (07/2009)
- Guidelines: Computing capital for incremental risk in the trading book (07/2009)
- Sound practices for backtesting counterparty credit risk models (04/2010)
- Basel III: A global regulatory framework for more resilient banks & banking systems (06/2011)
- Revisions to the Basel II market risk framework (02/2011)
- Principles for the Sound Management of Operational Risk (06/2011)
- Capitalisation of bank exposures to central counterparties - consultative document (11/2011)
- Fundamental review of trading book capital requirements - consultative document (05/2012)
- A framework for dealing with domestic systemically important banks (D-SIBs) (10/2012)
- Principles for effective risk data aggregation and risk reporting (01/2013)

Basel Committee published an unprecedented series of seminal papers.

Regulation goes Europe – European System of Financial Supervision (ESFS)



European Supervisory Authorities

- » European Banking Authority (EBA, London)
- » European Insurance and Occupational Pensions Authority (EIOPA, Frankfurt)
- » European Securities and Markets Authority (ESMA, Paris)

European regulatory publications

- » EBA publications concretise requirements defined in CRR and CRD IV
- » Publications divided into Regulatory Technical Standard and Implementation Technical Standard (RTS / ITS)

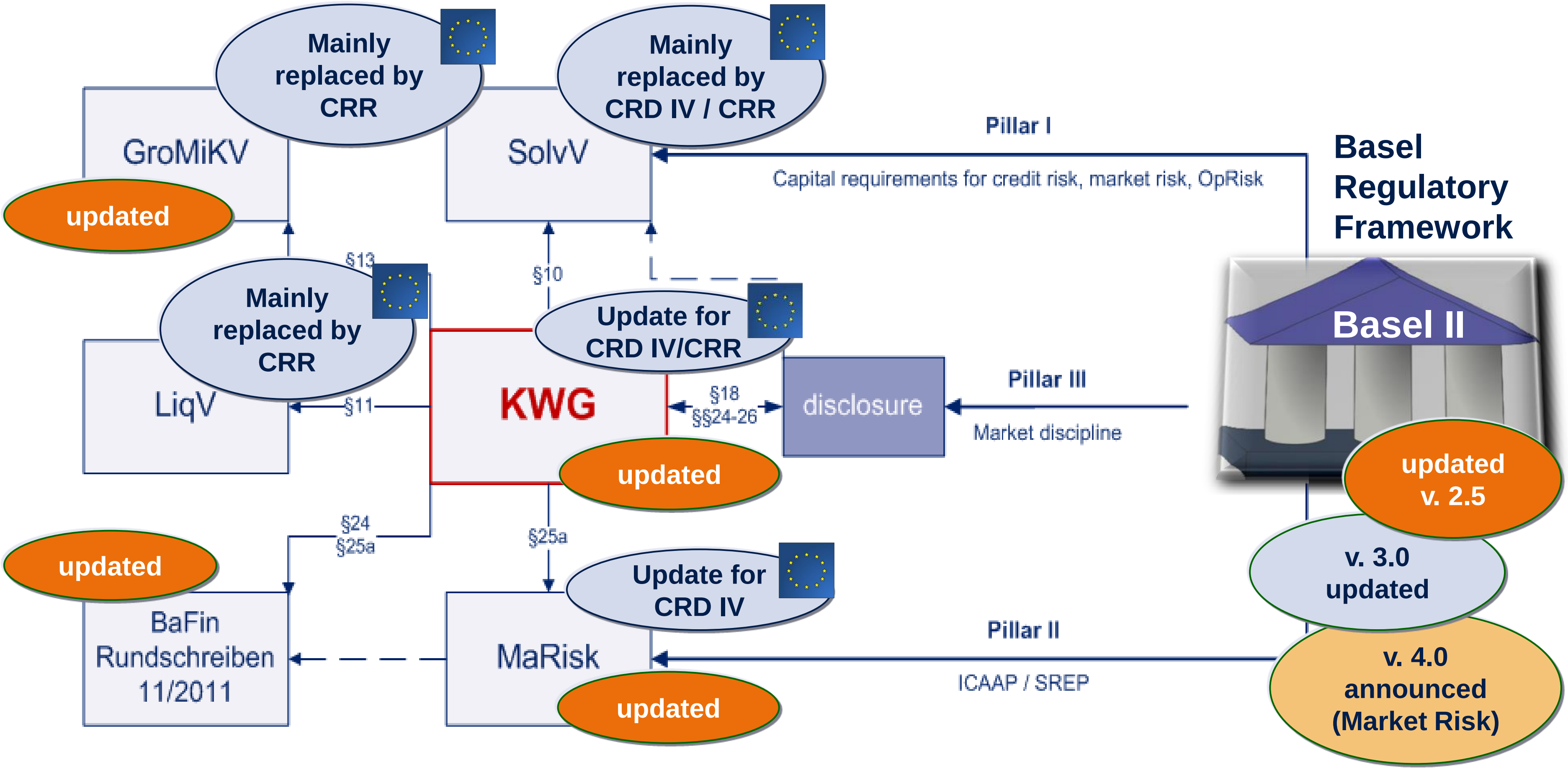
Regulations EU 1092-1095/2010 of European Parliament and Council

Recent selected publications

- » Consultation on guidelines on technical aspects of the management of interest rate risk arising from non trading activities (IRRBB) (EBA/CP/2013/23)
- » Consultation paper on RTS on prudent valuation (EBA/CP/2013/28)
- » Consultation paper on draft RTS on Capital Requirements for CCPs (EBA/CP/2012/08)
- » EBA consultation papers on guidelines to the Stressed Value At Risk (Stressed VaR)

A lot of national regulatory authority is now carried out by central authorities on European level under governance of the European Systemic Risk Board (ESRB), a sub-agency of the ECB.

Impact on German Regulation (as of 2014)



Nearly all German regulation has been reworked.

Standardisation of Capital Requirements – Structure of the CRR

Guideline: All relevant risks of a financial institution must always be backed by sufficient capital.

Part 1: General provisions

Part 2
Own Funds

Replace SolvV

Part 5
Exposures to transferred Credit

Part 6
Liquidity

Replace LiqV

Part 9
Implementation Acts

Replace SolvV

Part 3
Capital Requirements:

Title IV - Market Risk
Chapter 5 Internal Models

Title VI - CVA

Part 4
Large Exposures

Replace GroMiKV

Part 7
Leverage

Part 8
Disclosure

Replace SolvV

Part 10
Transitional Provisions

new SolvV

Part 11: Final Provisions

Capital Requirements Regulation defines Rules for Pillar I.

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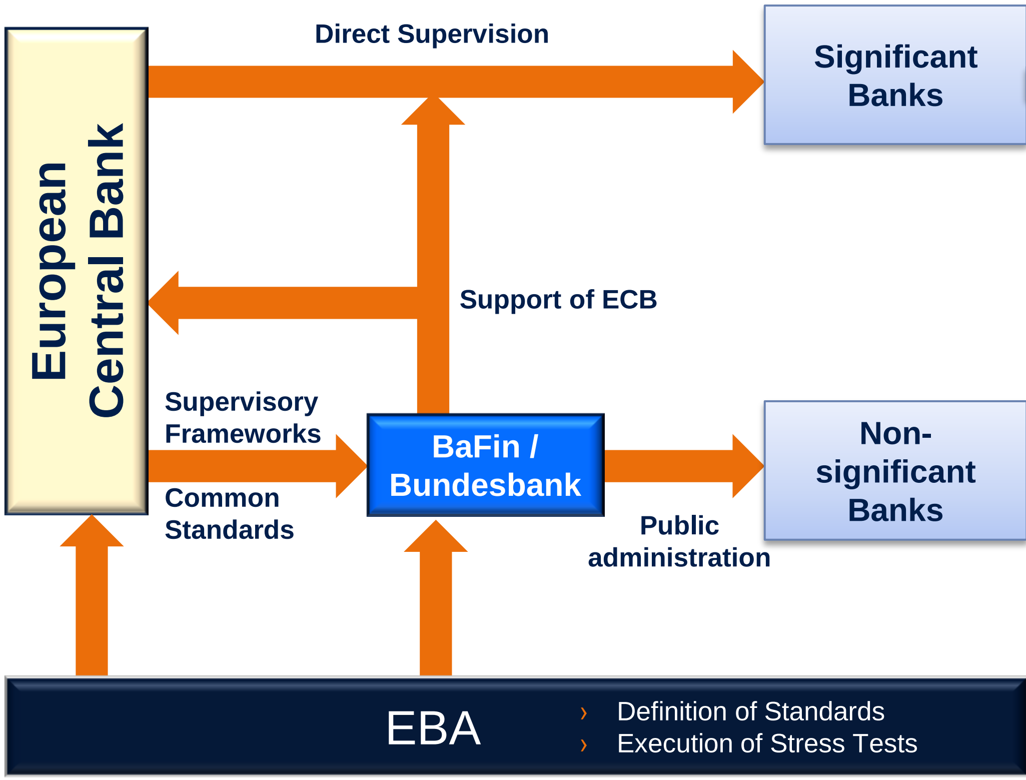
Stabilising the Euro-Zone – Single Supervisory Mechanism (SSM)



Significant Banks (SIBs)

Bank fulfils at least one criterion

Total assets > 30 bill. EUR	Total assets > 20% of GDP and Total assets > 5 bill. EUR	One of the three largest banks of the member state

Additional Criteria:

» Bank financially supported by EFSF/ESM
» ECB defines an international bank as SIB
» Bank voluntarily seeks supervision by ECB
» Voluntary participation in SSM from countries outside the Banking Union (Opt-in)

Euro zone supervision is organised as a Single Supervisory Mechanism (SSM) coordinated by the ECB. Its counterpart, the Single Resolution Mechanism (SRM) addresses the "Too Big to Fail" problem.

Each Model fits a Purpose

Capital requirements – Economic Capital vs Regulatory Capital

Regulatory Capital

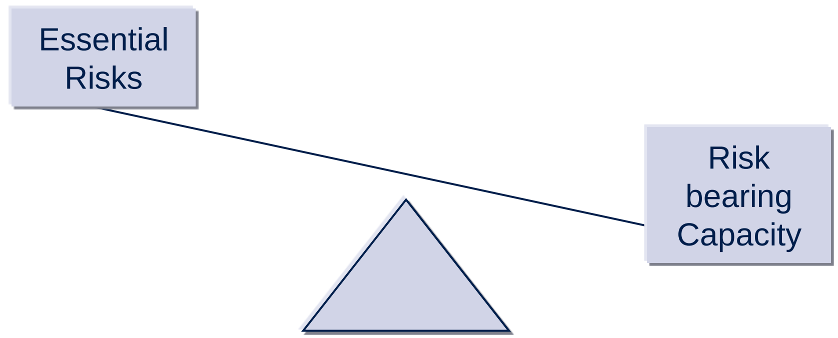
Minimum required by Pillar 1 of Basel III and CRR

	2014	2015	2016	2017	2018	2019
Additional Buffer for GSIBs						
Countercyclical Buffer			0-0,625%	0-1,25%	0-1,875%	0-2,5%
Capital Conservation Buffer			0,625%	1,25%	1,875%	2,5%
Total Capital	8%	8%	8%	8%	8%	8%
Tier 1	5,5%	6%	6%	6%	6%	6%
CET 1	4%	4,5%	4,5%	4,5%	4,5%	4,5%

Internationally harmonized approach, i.e. regulations are the same for all institutes

Economic Capital

Internal Capital Adequacy Assessment Process (ICAAP) required by Pillar 2 of Basel III



Methodological freedom for calculation of Risks and Risk bearing capacity

Proportionality Principle: regulations are adjusted based on the size and complexity of the institute

Both requirements need to be fulfilled for regulatory compliance.

Different Measures for different Purposes

There are different purposes in a bank that require market risk measurements:

	Pillar I	Pillar II	Internal Steering	…
Risk Measure Approach	**Regulatory Capital** **Standardised approach:** » Exposure based measure **Internal model approach:** » Risk based measure like VaR or ES	**Economic Capital** **Internal (Model) Approach:** » Risk based measure like VaR / ES or Stressed VaR » Stress Tests » Going concern /gone concern approach (or both) » Balance sheet or Value based	**Internal Limit System** **Internal (Model) Approach:** » Risk based measure like VaR or ES	…
Risk Model Parameterisation	Regulatory risk horizon of 10 days for internal model » Model risk horizon: usually 1 day; scaled by $\sqrt{10}$ » Time series used in model: typically 1-2 years » Confidence level: 99% » Backtesting of 1 day model	Risk bearing capacity » Model Risk horizon: usually 1 year » Time series: several years » Confidence level › Gone Concern: 99,9x% › Going Concern: 95%+	Management of market risks usually short term (trading book) » Risk horizon: usually 1 day » Time series: typically 0.5-2 years, weighted	…

Different purposes usually require different risk model parameterisations.

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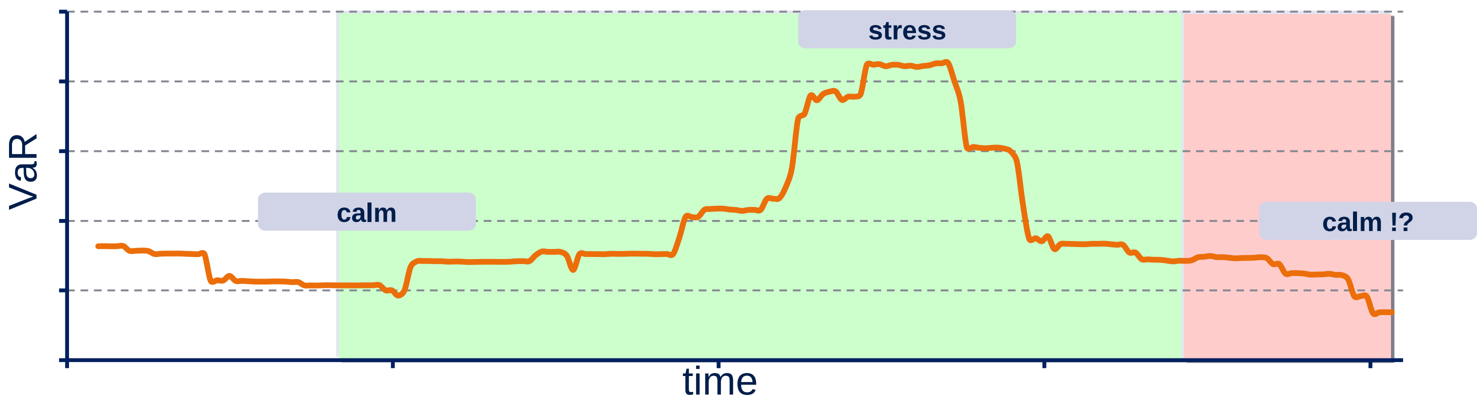
Different Measures for different Purposes

There are different purposes in a bank that require market risk measurements:

Pillar I / Regulatory Capital	Pillar II / Risk bearing capacity
Short term steering via VaR like models » Measurement and steering of „current" risk: possibly exp. weighted time series » Implies stationarity of current market phase » pro-cyclical risk measure	**Risk bearing capacity** » Stable risk measure / stable capital requirements based on complete economic cycle » Avoiding pro-cyclical capital requirements



Market risk depends on risk horizon and steering purpose.

Quantitative Modelling and Assessment of Market Risk

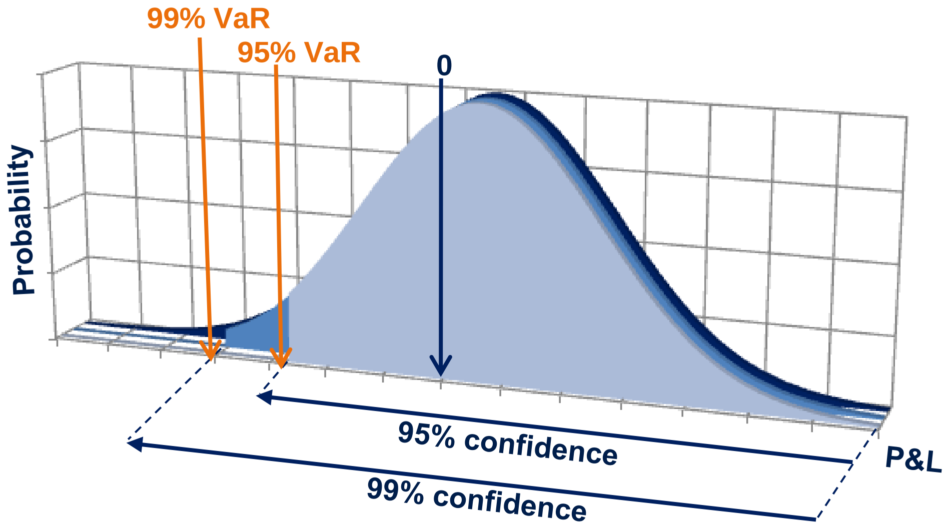
VaR Models

VaR Models – Prerequisites for Measuring Risk

Pre-requisites for Measuring Risk

- Include complete market risk position
- Ability of theoretical revaluation of all financial instruments of the portfolio
- Market data availability as basis for the revaluation
- Definition of risk factors
- Specification of a risk model
- Distribution of risk factors: assumptions and parameterization
- Mapping of risk factors to pricing models of instruments

Estimated Portfolio P&L Distribution



VaR is a widely used (standard) risk measure for risk management and the calculation of capital requirements within internal models.

VaR is a standard market risk measure for banks.

VaR Models – Overview of VaR Models

Calculating the P&L Distribution

The calculation of the **P&L distribution** on the **risk horizon** (usually 1 or 10 days) is the key element of each VaR model.

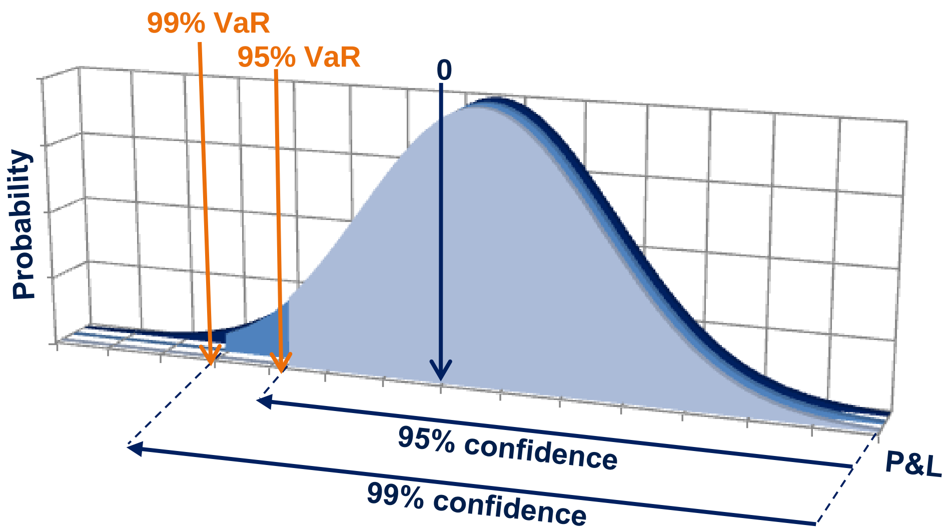
There a three different methods to calculate VaR established in the market:

» **Parametric VaR / Delta Normal VaR**

» **Monte Carlo Simulation VaR**

» **Historical Simulation VaR.**

Estimated Portfolio P&L Distribution



There are also hybrid models that combine e.g. historical and Monte Carlo simulation but we will focus on these three basic models.

We will concentrate on the model mechanics and leave out more technical issues like instrument aging, treatment of expiring instruments and cash flows over the VaR time horizon.

Standard VaR models: Parametric, Monte Carlo, Historical Simulation.

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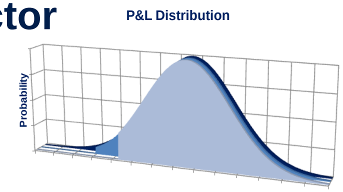
VaR Models – Parametric VaR / Delta-Normal VaR

Parametric VaR – Delta-normal VaR for an Instrument having a single Risk Factor



» **Assumption**: risk factor (log-)returns are normally distributed:

$$d\ln S(t) = \mu_s \cdot dt + \sigma_s \cdot dW \quad \text{with} \quad dW \sim N(0, dt)$$

instead we can also write:

$$dS(t)/S(t) = \tilde{\mu}_s dt + \sigma_s dW$$

change in S during dt is normally distributed random number

Quantile factor e.g. c=99% => Q≈2,326

short time horizon dt=1d/10d

neglecting $\tilde{\mu}_S$ $\quad \Rightarrow dS(t) \approx S(t) \cdot \sigma_s dW \xrightarrow[\text{quantile}]{\text{c-confidence}} Q_{1-c}^{N(0,1)} \cdot S(t) \cdot \sigma_s \sqrt{dt}$

» With approximation of relation between risk factor S and implied changes in value of risk position V being linear:

$$\Delta_s = \partial V / \partial S \Rightarrow \delta V \approx \Delta_s \cdot \delta S$$

» The delta-normal VaR is given by:

$$VaR \approx Q_{1-c}^{N(0,1)} \cdot \sqrt{dt} \cdot \sigma_s \cdot S(t) \cdot \Delta_s$$

Parametric VaR is based on Taylor-Expansion of pricing functions.

Simplest case: delta-normal VaR for single risk factor.

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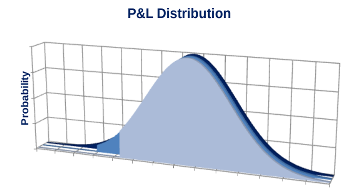
VaR Models – Parametric VaR / Delta-Normal VaR

Parametric VaR – Ingredients for an Extension to a multiple Risk Factor VaR



- Input for instrument / portfolio positions
  - Pricing model for each instrument
  - Instrument sensitivities Δ_i for each risk factor
  - Portfolio sensitivities as sum of instrument sensitivities of the portfolio
- Input for risk factors
  - **Volatility** σ_i for each risk factor
  - **Correlation** ρ_{ij} between risk factors i,j

Covariance matrix Σ

$$\Sigma = \begin{pmatrix} c_{11} & c_{12} & \cdots & \cdots & c_{1n} \\ c_{21} & \ddots & & ⋰ & c_{ij} \\ \vdots & & c_{ij} & & \vdots \\ \vdots & ⋰ & & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & \cdots & c_{nn} \end{pmatrix} \text{ with } \underset{\text{covariance}}{c_{ij}} = \overset{\text{correlation}}{\rho_{ij}} \sigma_i \sigma_j$$

The covariance matrix contains all the stochastic information the parametric VaR is based on.

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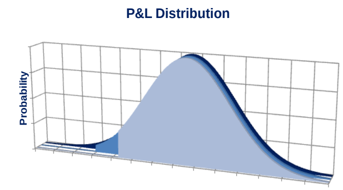
VaR Models – Parametric VaR / Delta-Normal VaR

Parametric VaR – Delta-normal VaR for multiple Risk Factor Portfolio V



» **Assumption**: risk factor (log-)returns are **normally distributed**:

$$d \ln S_i(t) = \mu_{S_i} dt + \sigma_{S_i} dW \qquad \text{with} \qquad dW \sim N(0, dt)$$

neglecting μ_S $\Rightarrow \delta S_i(t) \approx S_i(t) \cdot \sigma_{S_i} dW$

Change of portfolio value is sum of normal distributed random numbers

» **Linear approximation** of pricing functions: $\Delta_{S_i} = \partial V / \partial S_i \Rightarrow \delta V \approx \sum_i \Delta_{S_i} \cdot \partial S_i$

» The variance of the sum of normal distributed random numbers is given by sum of their covariances:

$$\begin{aligned} \operatorname{var}[\delta V] &\approx \sum_{i,j=1}^{n} \Delta_{S_i} \Delta_{S_j} \operatorname{cov}[\delta S_i, \delta S_j] \\ &= \sum_{i,j=1}^{n} \Delta_{S_i} \delta \Sigma_{ij} \Delta_{S_j} \\ &= dt \sum_{i,j=1}^{n} \Delta_{S_i} S_i \sigma_i \rho_{ij} \sigma_j \Delta_{S_j} S_j \end{aligned}$$

This implies the delta-normal VaR given by:

$$VaR \approx Q_{1-c}^{N(0,1)} \sqrt{dt} \sqrt{\sum_{i,j=1}^{n} \Delta_{S_i} S_i \sigma_i \rho_{ij} \sigma_j \Delta_{S_j} S_j}$$

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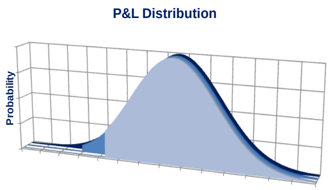
VaR Models – Parametric VaR / Delta-Normal VaR

Parametric VaR – Pro and Contra



Advantages

- Simple
- Fast
- Reproducible (analytical formula)

Disadvantages

- Non-linear effects not captured
- Assumption of normal distribution
- Covariance matrix as necessary input

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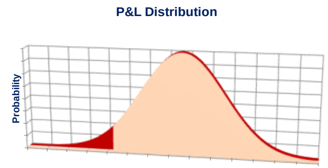
VaR Models – Monte Carlo VaR

The calculation of the Portfolio P&L distribution on the risk horizon is the basis of each VaR model.



- **Monte Carlo Simulation – Idea**
  - Instead of an analytic approximation, as in the case of the parametric VaR , the P&L distribution is generated by revaluation of the portfolio under a large number of risk factor scenarios.
  - The risk factor scenarios are generated from the assumed risk factors distributions / stochastic processes.
- **Example (log-normal distribution):**

time step **volatility**

drift **random number**

relative risk factor change

$$r_{\log}(t,\delta t) = \mu \delta t + \sigma \sqrt{\delta t}\, X$$

new risk factor value

$$S(t+\delta t) - S(t) = \delta S(t) \approx S(t)\left(e^{r_{\log}} - 1\right)$$

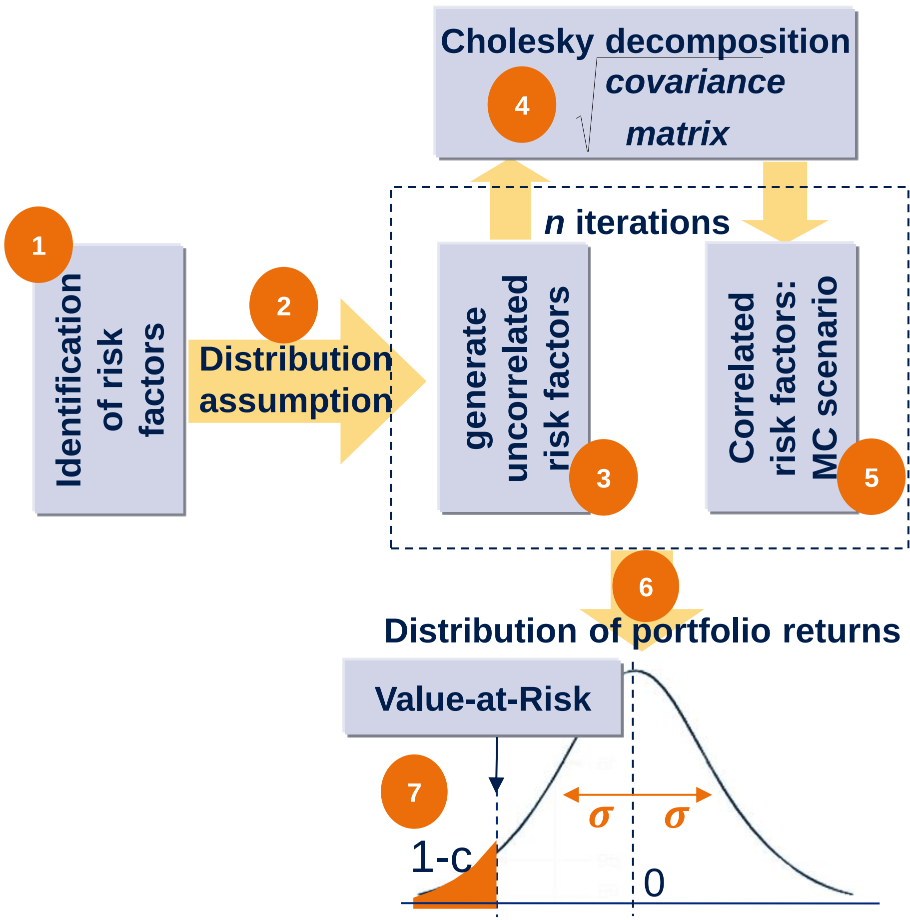
old value **relative change**

Basic idea: generate P&L distribution from large number of scenarios.

VaR Models – Monte Carlo VaR

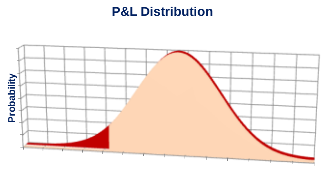
Illustration of main Calculation Steps

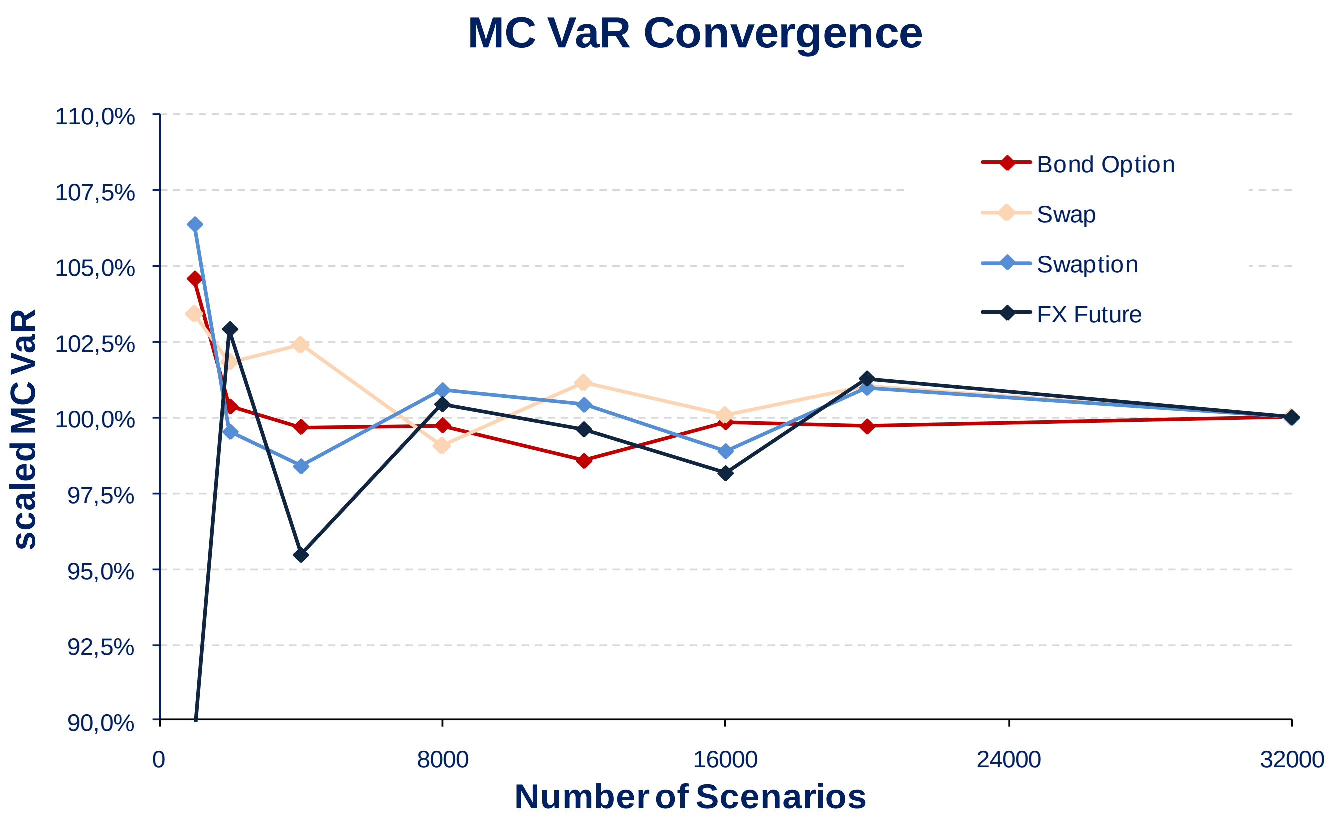
- **(1)** Identification of all relevant risk factors
- **(2)** Distribution assumption(s) for risk factor returns over the time horizon from time series analyses
- **(3)** Generate random numbers for the uncorrelated risk factor returns according to the distribution assumption(s)
- **(4)** Transformation of uncorrelated risk factors into correlated risk factors via multiplication with Cholesky (or e.g. SVD) decomposition of covariance matrix: correlated risk factors returns form a Monte Carlo scenario.
- **(5)** Full (or sensitivity based) revaluation of portfolio under the generated scenario
- **(6)** Generation of a large number of scenarios leads to P&L distribution
- **(7)** Estimation of VaR as the desired quantile of this distribution



VaR Models – Monte Carlo VaR

Example of MC VaR for different Instruments as Function of No. of Scenarios

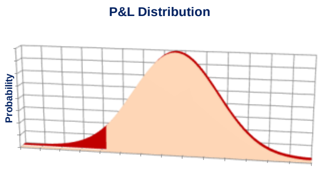




Trade-off between precision and performance.

VaR Models – Monte Carlo VaR

Monte Carlo VaR – Pro and Contra



Advantages

- Considers non-linear risk
- No approximations necessary
- VaR can be assigned to a certain scenario
- Simulated time horizon can be chosen (given stochastic processes for risk factors)

Disadvantages

- Time consuming
- Decomposition of VaR (e.g. into interest rate, equity, FX VaR) only with new simulation
- Statistic errors, only to some extent reproducible
- Covariance matrix as necessary input
- Usually normal distribution assumed

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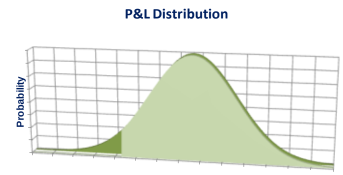
VaR Models – Historical Simulation VaR

The calculation of the Portfolio P&L distribution on the risk horizon is the basis of each VaR model.



- **Historical Simulation – Idea**
  - Similar to the Monte Carlo Simulation the historical simulation tries to generate the Portfolio P&L distribution from revaluation of the portfolio under a large number of risk factor scenarios.
  - The risk factor scenarios are directly calculated from the observed historical risk factor returns. Therefore no distribution assumption for the risk factors is required (only the return type has to be specified).

- **Example (log-return):**

time step: 1d

relative risk factor change

$$r_{\log}(t, \delta t) = \ln \frac{S(t = 01.10.2010)}{S(t = 30.09.2010)}$$

new risk factor value

$$S(t + \delta t) - S(t) = \delta S(t) \approx S(t)\left(e^{r_{\log}} - 1\right)$$

old value

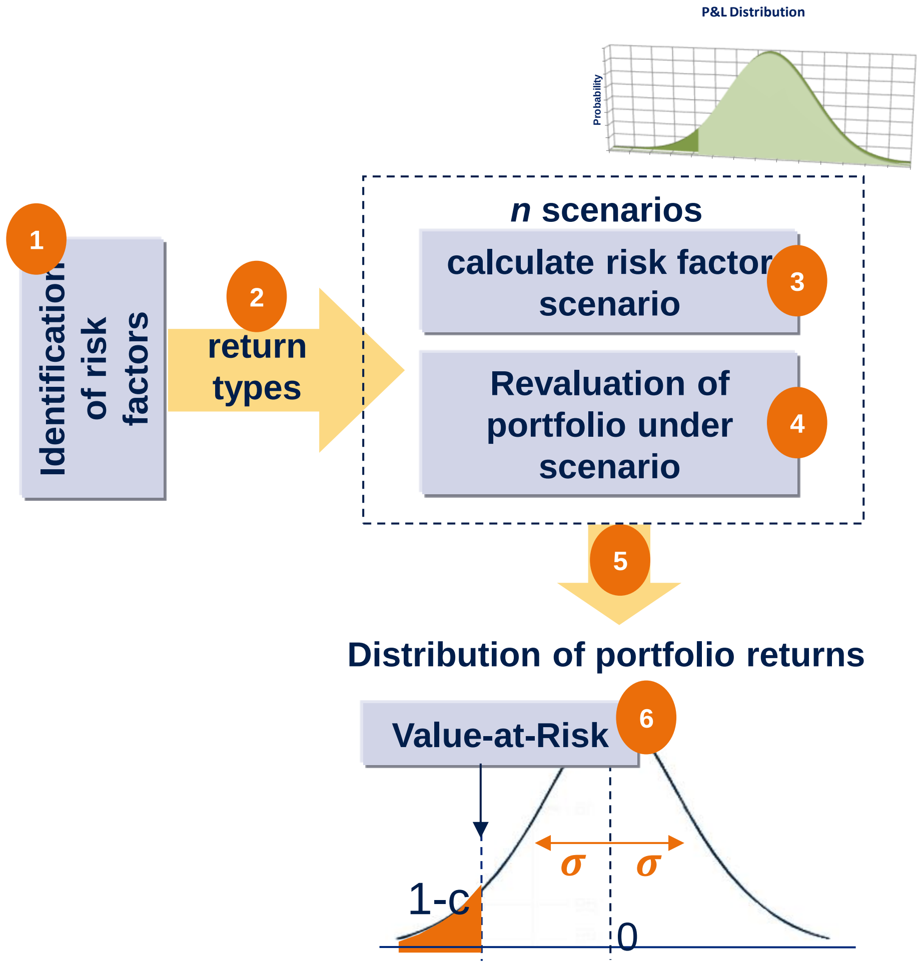
relative change

Basic idea: generate P&L distribution from large number of historical scenarios.

VaR Models – Historical Simulation VaR

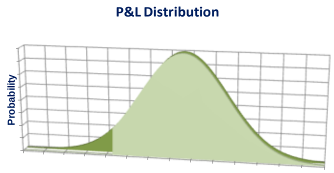
Illustration of main Calculation Steps

- **(1)** Identification of all relevant risk factors
- **(2)** Specification of return type for each risk factor
- **(3)** Calculate scenario from observed risk factor returns
- **(4)** Full (or sensitivity based) revaluation of portfolio under historical risk factor scenario.
- **(5)** Repeat **(3)** and **(4)** for all historical scenarios (typically 250 or 500)
- **(6)** Estimation of VaR as the desired quantile of this distribution



VaR Models – Historical Simulation VaR

Historical Simulation – Differences to the other Models



- » Risk factor return scenarios are not generated from stochastic processes, but are based on **observed historical risk factor returns**.
- » As a consequence: **no distribution assumption / stochastic process** is needed for the risk factors.
- » Instead of using the covariance matrix, the entire time series of risk factor returns are taken into account.
- » Volatility and correlation are not estimated explicitly, but are used indirectly.
- » For historical simulation market data quality is of paramount importance.
- » Compared to Monte Carlo Simulations only a small number of scenarios (typically 250 or 500) are used. Therefore the statistical error bars are much bigger.
- » If time series are long enough the historical simulation can account for fat tails.

Historical simulation needs no risk factor distribution assumptions.

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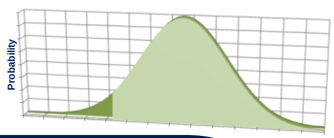
VaR Models – Historical Simulation VaR

Historical Simulation VaR – Pro and Contra



Advantages

- No assumption about risk factor return distribution necessary (esp. no assumption of normal distribution)
- Non-linear risk is considered
- No approximations necessary
- VaR can be attributed to a certain scenario (incl. date)

Disadvantages

- Strong dependency on quality of time series
- Difficult to calculate VaR for long time horizons
- Only small number of scenarios possible (according to length of time series), implies jumps in results
- Decomposition of VaR (e.g. into interest rate, equity, FX VaR) only with new simulation

VaR Models – Comparison of VaR Methods

Comparison of VaR Methods

	Parametric VaR	Monte Carlo Simulation	Historical Simulation
Coverage of non-linear risks	−	+	+
Assumption of distribution of risk factors	−	−	+
Speed / time consumption	++	−	+
Size of statistical samples	+	+	–
VaR attributable to scenario	−	+	++
Need for explicit volatilities and correlations	−	–	+
Dependence on time series quality	+	+	– –
Decomposition of VaR	+	–	–
Extension by new risk factors	+	+	–

Each VaR method has its advantages and disadvantages.

VaR Analyses

Analyses of VaR – a Number in Detail

Motivational issues:

» By itself, VaR is just a number

» What can we learn from this number?

» Is a decomposition possible?

» How can VaR be interpreted?

» Which kind of risk management and control impulses can be deduced from VaR?

» To which extent does the VaR methodology influence these analyses?

Possible approaches:

» Statistical decomposition
(direct or via related measures)

» Dynamical point-in-time-analyses

» Validation techniques

Regulatory requirements

» **MaRisk:**
  › **identification of risk concentrations, supervision of limits, backtesting**

» **Capital Requirements Regulation (CRR) §369:**
  › **daily analysis of VaR**
  › **regular periodic validation**
  › **backtesting**

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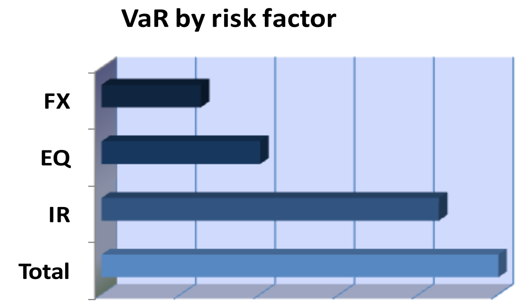
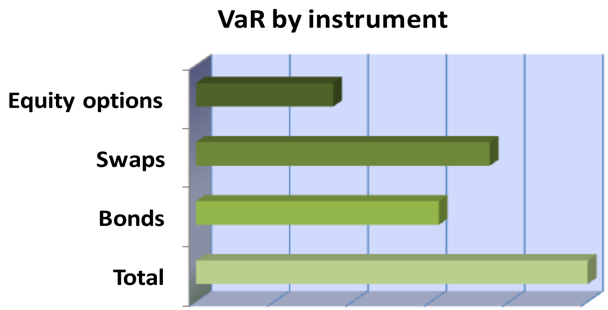
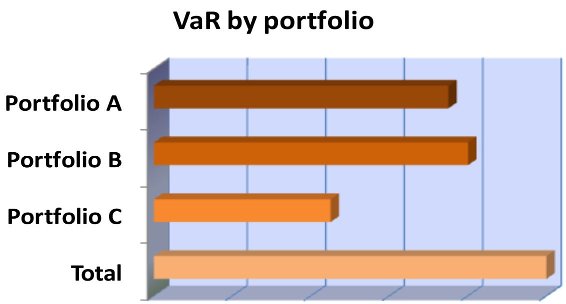
Origins of Risk – Statistical Decomposition

Making VaR plausible – a recipe:

- » Decompose VaR for a given trade day and its statistical conditions, i.e. keep portfolio position and market data fixed to identify sources of risk
- » Criteria for decomposition (may be combined):
  - › trade categories:
    - › portfolios and instrument classes
    - › industries, regions and issuers
  - › risk factor categories:
    - › equity, foreign exchange
    - › interest rates, spreads
- » Necessary requirement:
  access to aggregation categories on trade level
- » Alternative: VaR can be decomposed via related risk measures







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Extended Statistical Analysis – Statistical Decomposition

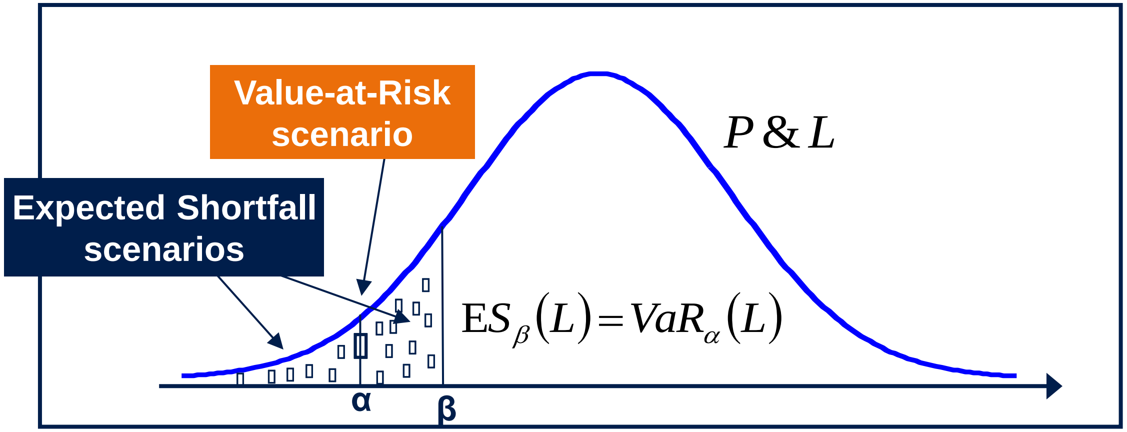
Expected Shortfall Approach

- » VaR lacks statistical information beyond its corresponding scenario
- » Direct decomposition may prove unstable

Recipe

- » Relate VaR to Expected Shortfall by quantile transformation
- » Stabilise and linearise risk contributions
- » Apply decomposition ansatz to Expected Shortfall figures

Value-at-Risk (quantile α)	$\mathrm{VaR}_{\alpha}(L) \equiv \inf\{x; \mathrm{P}(L > x) \le 1-\alpha\}$
Expected Shortfall (qu. β)	$\mathrm{ES}_{\beta}(L) \equiv E(L \mid L > VaR_{\beta}(L))$



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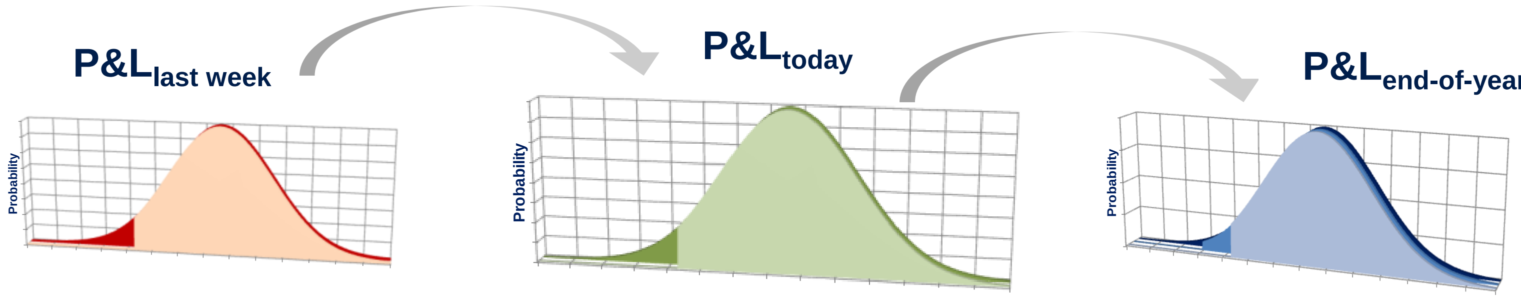
Comparing the Difference – Dynamical Points-in-time Analysis

Motivation

- Explains and illustrates the changes in VaR between different points in time
- Dynamical analysis is either forward- or backward-looking



Backward-looking: cause analysis

- Explain VaR-changes caused by changes in
  - position, valuation/statistical parameters, scenarios
- Investigation of errors in input data and valuation
- Focus on most dominant changes in components

Forward-looking: evolutionary analysis

- Incremental and marginal VaR
- “What-If”-analyses: Changes in VaR caused by:
  - influence of market data or statistical parameters
  - new (hypothetical) risk position,
e. g. pre-deal check (new simulation needed)

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Cause analysis – Dynamical Points-in-time Analysis

Motivation

» Stepwise reconciliation of reference and current VaR

Derivation of original VaR

Reconciliation Tool

Derivation of VaR with updated risk position

Derivation of VaR with updated valuation parameters

Derivation of VaR with updated statistical parameters*

Exchange of step order possible (leads to different intermediate results)

*** e. g., covariance matrix or exchange of scenarios in historical simulation**

» High performance demands on infrastructure

› derivation of position delta

› revaluation with adapted parameters

Model Validation

Extended Investigation – Validation of VaR Model

Validation of VaR – Possible Ansatzes

- Validation is strictly speaking no analysis, but provides insight into quality of the model
- Indicates starting points for the analysis of VaR, particularly when investigating unexpected changes
- Allows for investigation and detailed check of the parameterisation (may depend on the VaR model):
  - **Sensitivity-based models:**
    - Which sensitivities enter the simulation? Deviation from full-valuation?
    - How are sensitivities calculated (one/two-sides, step width, etc.)?
  - **Simulation-based models:**
    - Sufficiently stable statistics? Assumptions of distribution correct?
    - Influence of various return types of risk factors
- Is the usage of consistent market data guaranteed? Are historical data series quality assured?
- Analysis of full P&L-distribution contributes hints concerning model stability
- Are all relevant risk factors taken care of?

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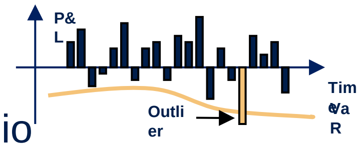
Regulatory Required Validation – Back Testing

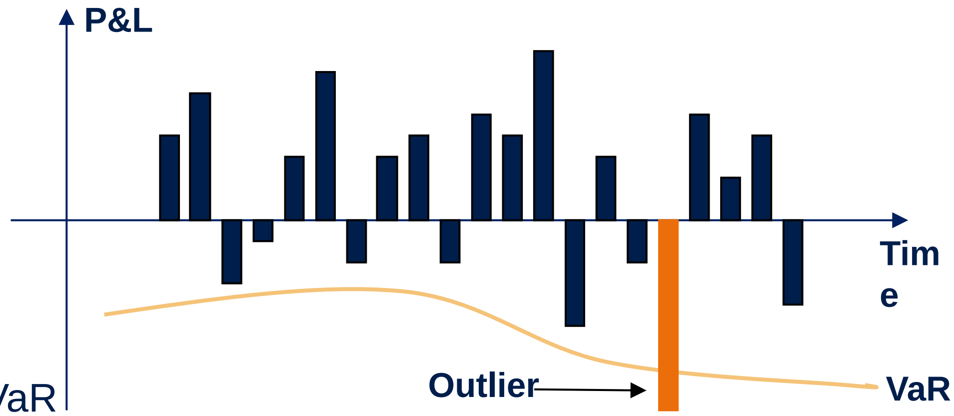
Back Testing VaR Models – Basic Idea



- VaR makes a probabilistic statement about the change in value of a portfolio over the risk horizon.
- A validation of this statement is only statistically possible – by **comparison of the VaR prediction with the actual P&L realisation**:

VaR(t,t+dt) *vs.* ***PV(t+dt) – PV(t)*** with *dt = 1d*

- This comparison is called **back testing**
It is a very useful tool and is regulatory required by **MaRisk** and **CRR**
- **Aside:** Since back testing is performed on a day-to-day basis, the regulatory VaR with a holding period of 10 days has to be either properly transformed or a one-day-VaR has to be additionally calculated.



Back testing: statistical validation of risk model.

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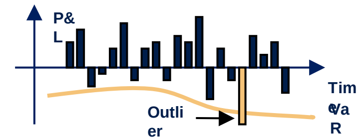
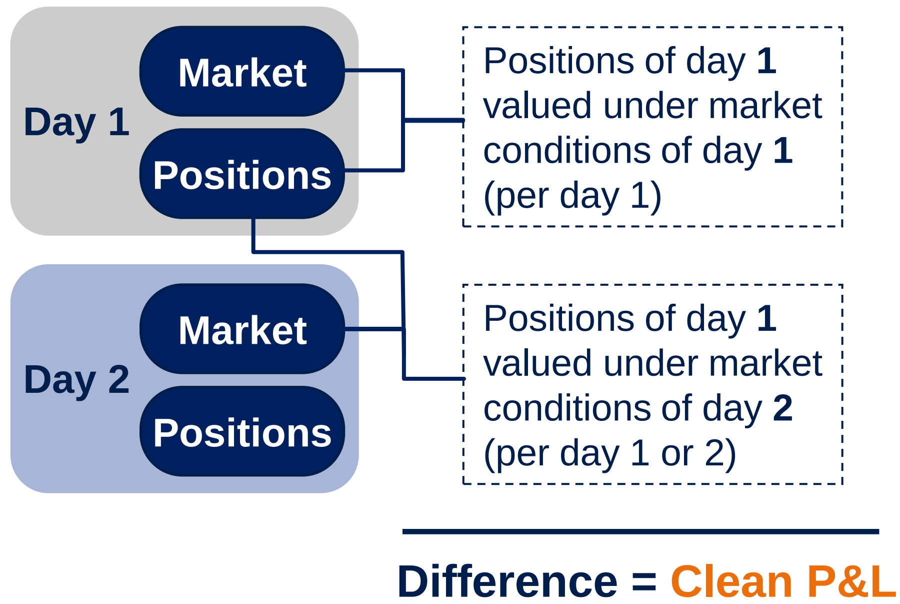
Clean and Dirty – Back Testing

Back Testing VaR Models – Clean Back Testing

- Back testing should only take into account effects that are part of the risk model. Therefore position changes over the risk horizon or subsequent data corrections should be reverted in the calculation of the actual P&L for comparison.
  This kind of P&L is called **clean P&L**.
- Back testing based on the clean P&L is called **clean back testing**, in contrast to **dirty back testing** where position changes and data corrections influence the P&L.
- Back testing should be based on market-quoted present values as extensively as possible. This procedure is called **mark-to-market**.
- If data is not available, a **mark-to-model** may be used under certain constraints





If possible, clean back testing should be used for validating a VaR model.

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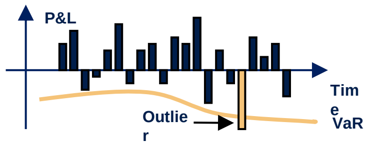
Outliers and Model Quality – Back Testing



From the Number of Outliers to a Judgement of Model Quality

- » Model OK if number of outliers within "usual" **random** Fluctuations
- » Which deviation from expected number of outliers is still **random**?
- » **Statistics** ⟶ **Hypothesis Tests**
  - › if there are "too many more" outliers than expected than
    - › deviations are judged significant
    - › something is probably wrong with the model
  - › in our case two possible outcomes per back testing period: either P&L is within VaR limit or not
    - › therefore significance of deviation judged by **binomial test**

There are also many other tests possible

If possible, clean back testing should be used for validating a VaR model.

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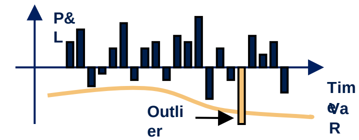
Binomial Distributions – Back Testing



Binominal Test of Back Testing Time Series

» Parameter
p = probability for each single outlier (expected to be e.g. 1%)
k = number of outliers
n = number of back testing periods

Binomial distribution:

» Probability of **exactly** *k* outliers:
(if model is correct)

$$b(k,n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$

» Probability of *k* or less outliers:
(if model is correct)

$$\sum_{i=0}^{k} b(i,n,p)$$

» probability of type I error (rejection of correct model)
(if rejected for k or more outliers)
equals probability for k or more outliers:
(if model is correct)

$$1 - \sum_{i=0}^{k-1} b(i,n,p)$$

» probability of **type II error** (acceptance of wrong model) needs additional assumption about the “wrong model”

Binominal test for back testing of VaR model.

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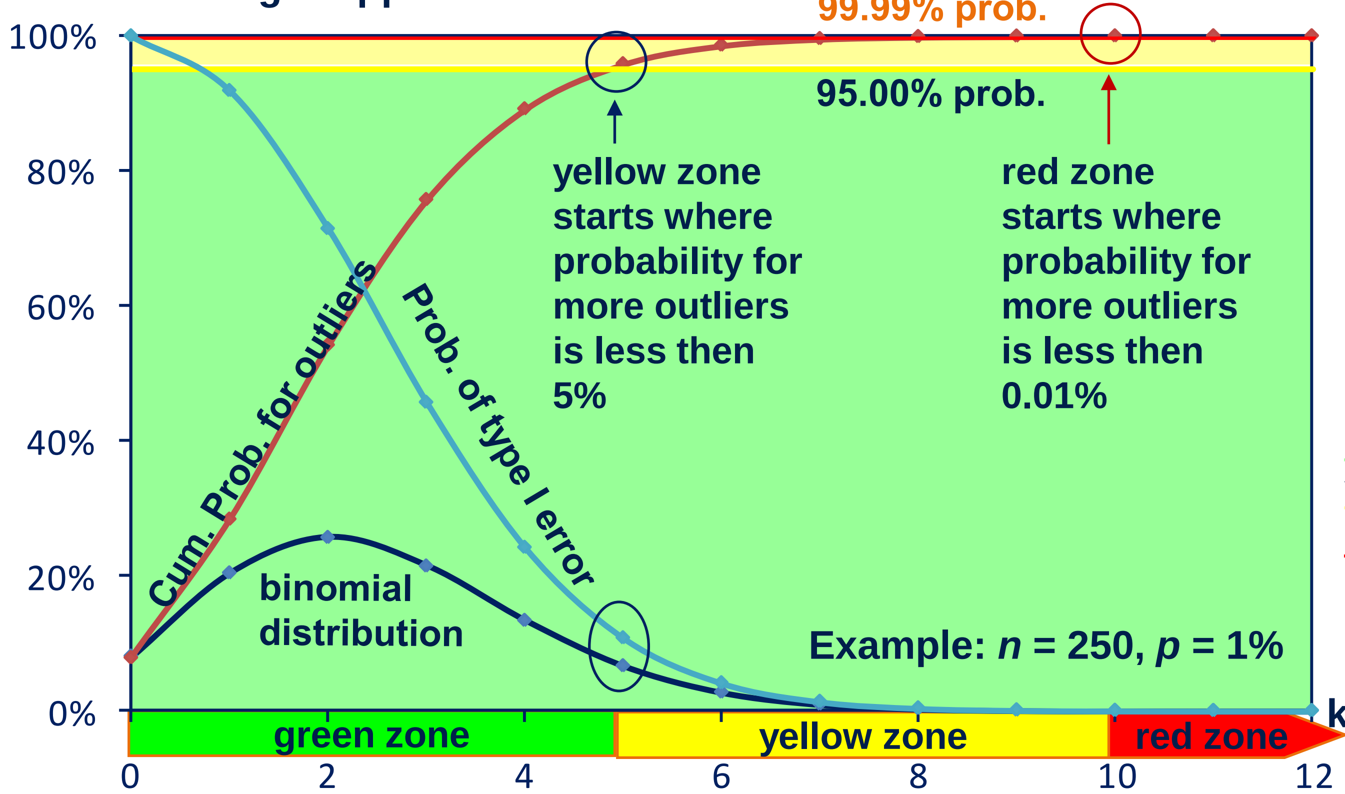
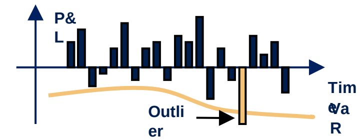
Back Testing and Stress Testing – Back Testing

The Traffic Light Approach





F1(k): type I error reject correct model for k outlier

green: F1(x) > 5%
yellow: F1(x) > 0,01%
red: F1(x) < 0,01%

The traffic light approach is used by the regulator to check VaR models.

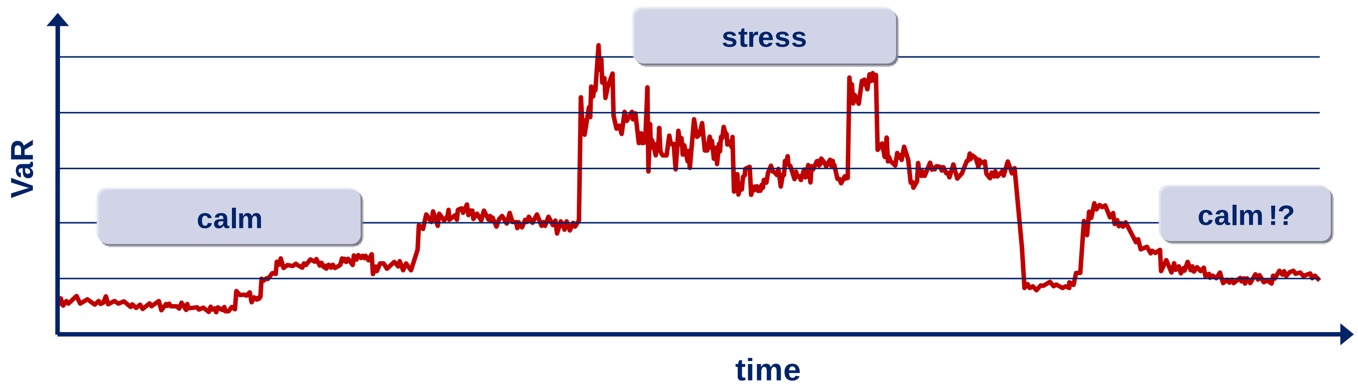
Extended Risk Assessment

Stressed VaR

Countering Pro-Cyclicality – Stressed VaR

Motivation

» VaR is usually parameterized on (weighted) market data of one or two fiscal years, and therefore insensitive to events of earlier periods. Thus, VaR is **pro-cyclical**, i. e. small in face of calm market and large in periods of stress

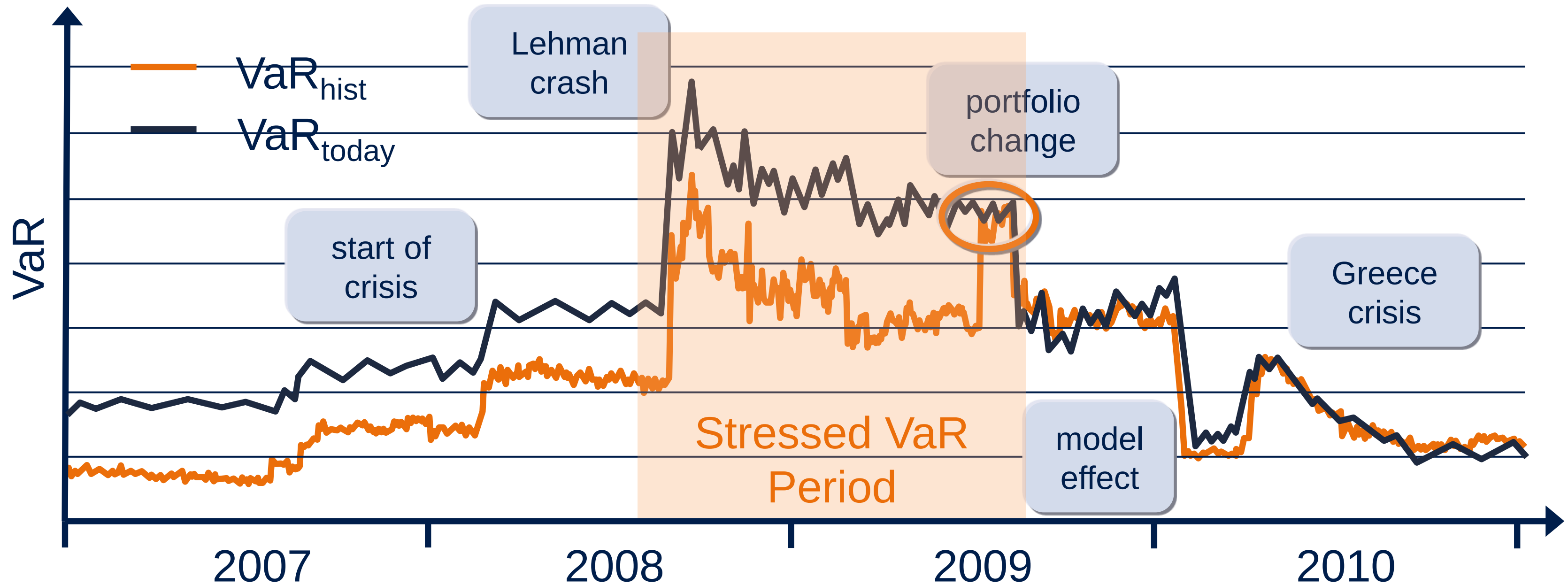


» With a focus on **regulatory capital,** banks solely relying on VaR are not thoroughly prepared for a jump-into-crisis, since capital requirements have been (extremely) low beforehand and capital can not be easily raised in a critical market environment

» An ansatz to counter this drawback is the concept of **Stressed VaR**, where the standard VaR is calculated for the momentary portfolio positions, but is parameterised for a **suitable** historically observed **one-year-period of stress**

The Theoretical Ansatz – Stressed VaR Period

» Calculate a pseudo-historical VaR series based on the preceding one year of market data for the current portfolio. Identify local maxima, compare with risk and business strategy of the institute, and choose the suitable drastic period



» Perfect ansatz in theory, but in general not viable due to technical performance issues, impossible or misleading mapping of risk factors

Stresstesting

Insight beyond VaR – Stress Testing

Stress Testing – Basic Idea

» Stress testing is related to the simulation methods for calculating VaR. In both cases the portfolio is revaluated under scenarios of changed risk factor values.

» A stress test scenario is given by the “manual” specification of changes for all relevant risk factors.

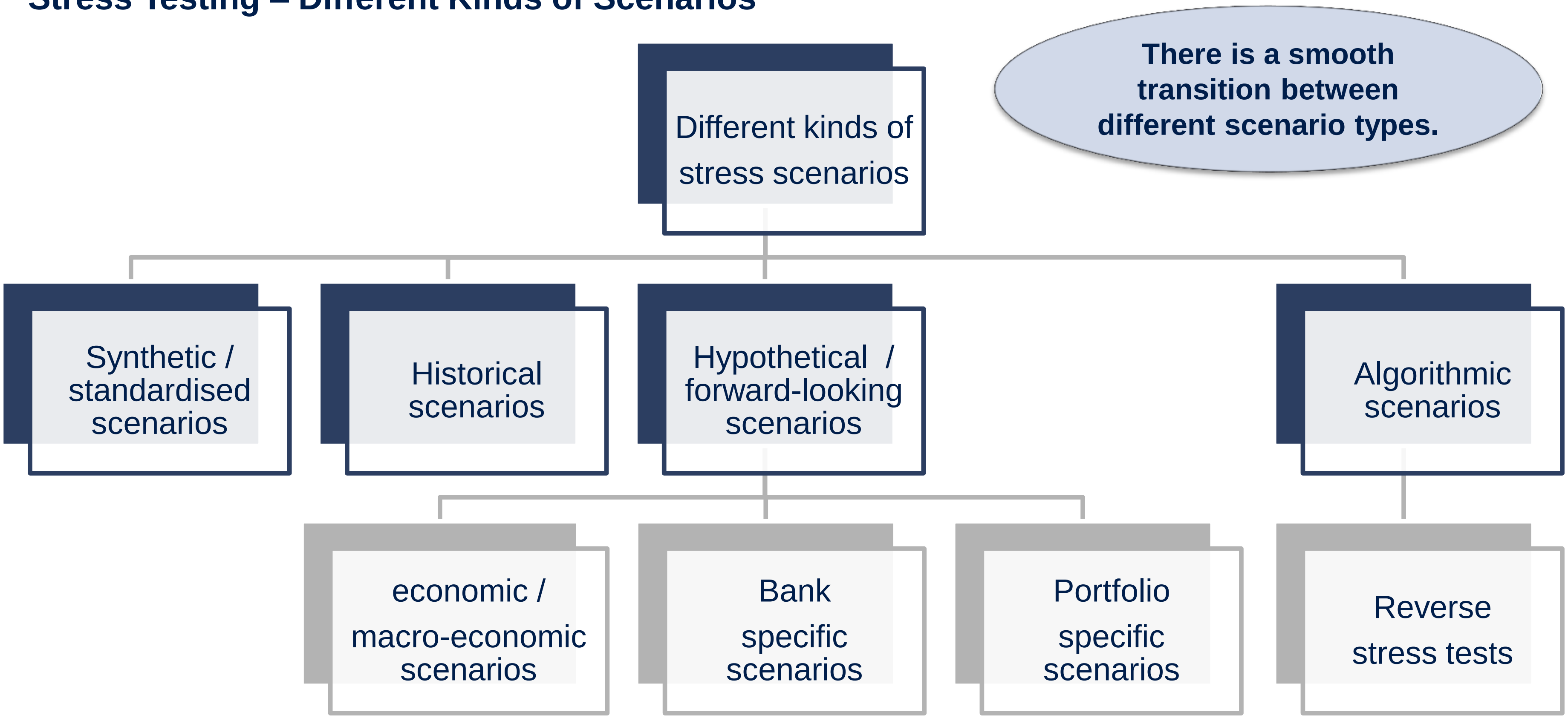
» **Stress testing complements VaR** and enables a different view on the risks of a portfolio.

» Basic steps:
  › Define the scenario.
  › Perform a full revaluation of the portfolio for this stress scenario.
  › The difference between this calculated value and the current portfolio value is the profit or loss resulting from the stress scenario.

Stress tests are used to complement VaR.

Severe but Plausible – Stress Testing

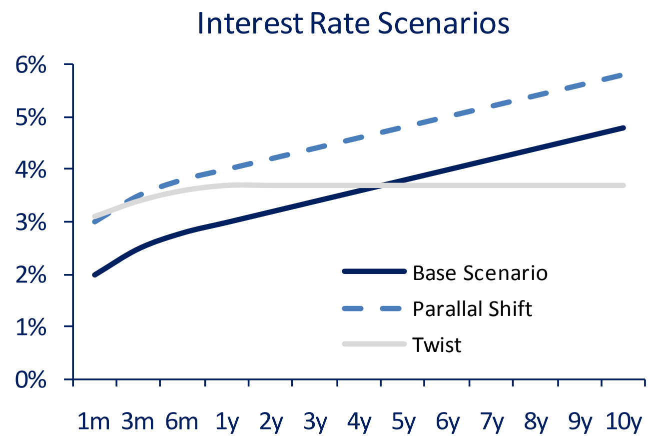
Stress Testing – Different Kinds of Scenarios



Back Testing and Stress Testing – Stress Testing

Examples for Stress Test Scenarios

» **Synthetic scenarios**
- › Interest rate: parallel shift of the interest rate curve of +/- 100 bp
- › Interest rate volatility: shift of +/- 20 %
- › Equity: shift of share prices and index prices of +/- 10 %
- › Equity volatility: shift of +/- 20 %
- › FX: shift of all exchange rates of +/- 6 %
- › Non-parallel interest rate curve shifts (tilt, butterfly, ...)



» **Historical scenarios**
- › Recent financial crisis
- › Asia crisis, 9/11, etc.
- › Define appropriate shifts representing the historical scenario if historical data can not be fed into the pricing system or is not available (e.g. price shift for all US shares, shock for interest rates of certain currencies)

Ideal Coverage in the Eye of the Regulator – Stress Testing

Sound stress testing includes...

- Counterparty and contractual risk
- Historical, sophisticated & forward-looking scenario engineering
- Full coverage of group-wide risks
- Challenge of risk mitigation strategies
- Focus on complex products
- Inverse scenario generation
- Consideration of reputation risk
- Consideration of refinancing risk
- Coverage of positions beyond the balance sheet
- Consecutive risks and stress of correlations
- Coverage of risk factors not-in-VaR

As part of a comprehensive approach, stress testing has a leading role to play in strengthening bank corporate governance and the resilience of individual banks and the financial system.

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Practical Issues of Market Risk Controlling

A Model alone is no good

At first glance, the mathematical model seems to be the most difficult challenge in measuring market risk.

Still, there are a lot of questions to pose and things to worry about:

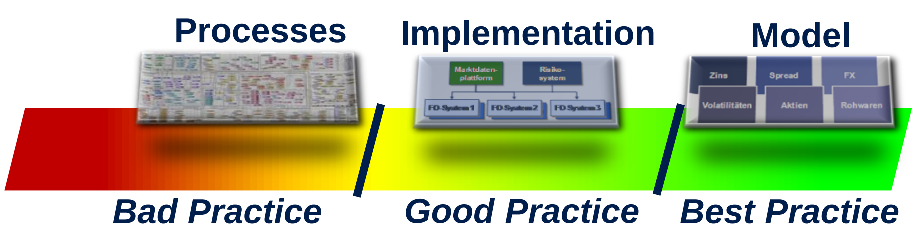
» Is data coverage and quality sufficient?

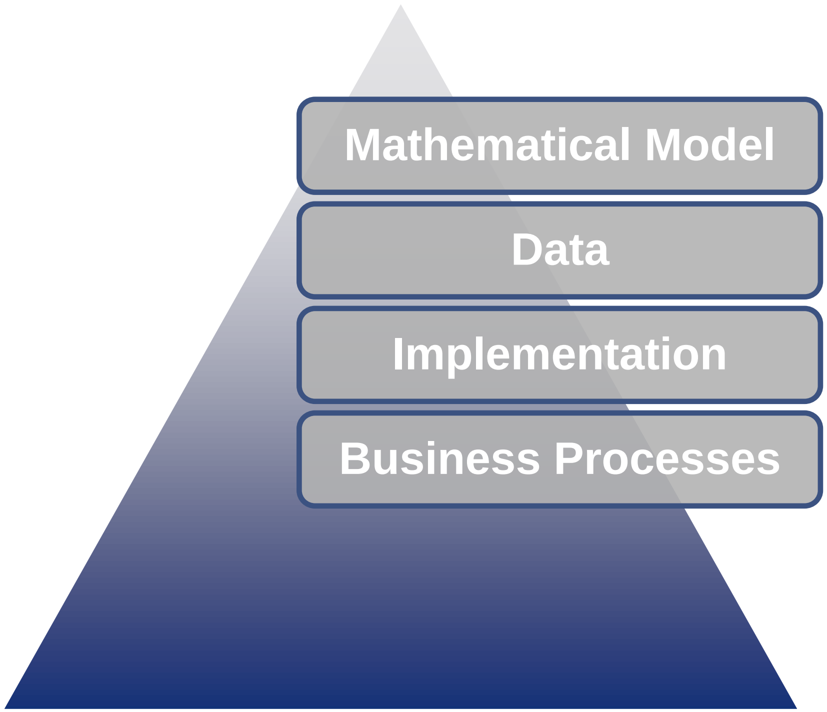
» Does the model cover all risk positions?

» Are IT systems/ processes stable and efficient?

» Are roles and responsibilities adequately defined?

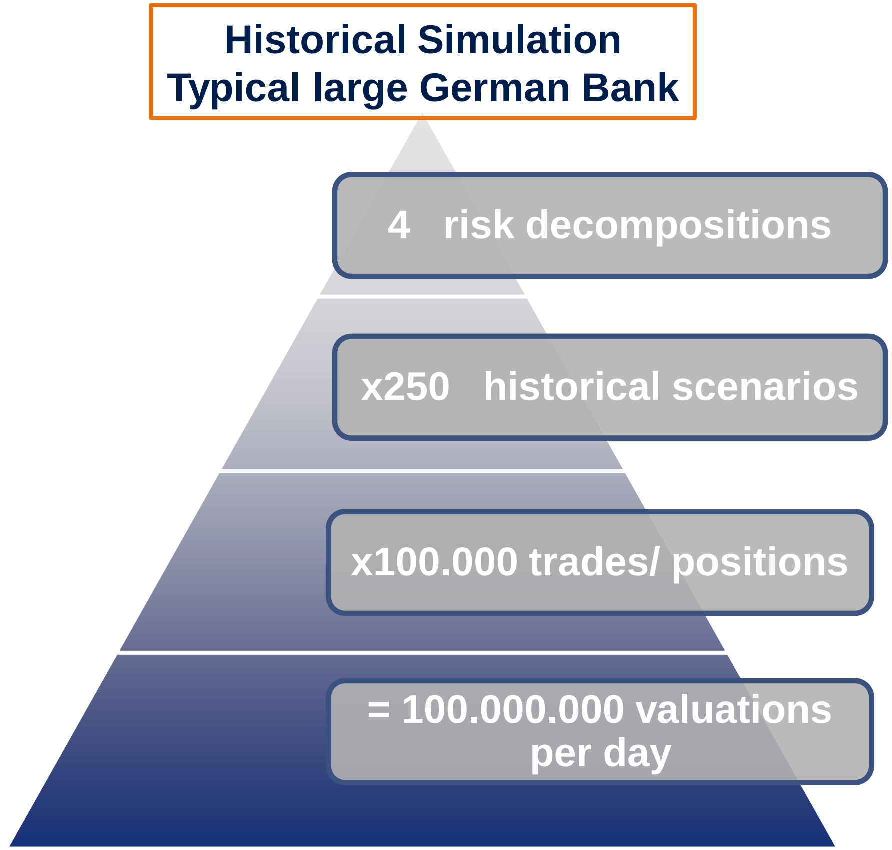
» Do we comply with regulatory requirements?



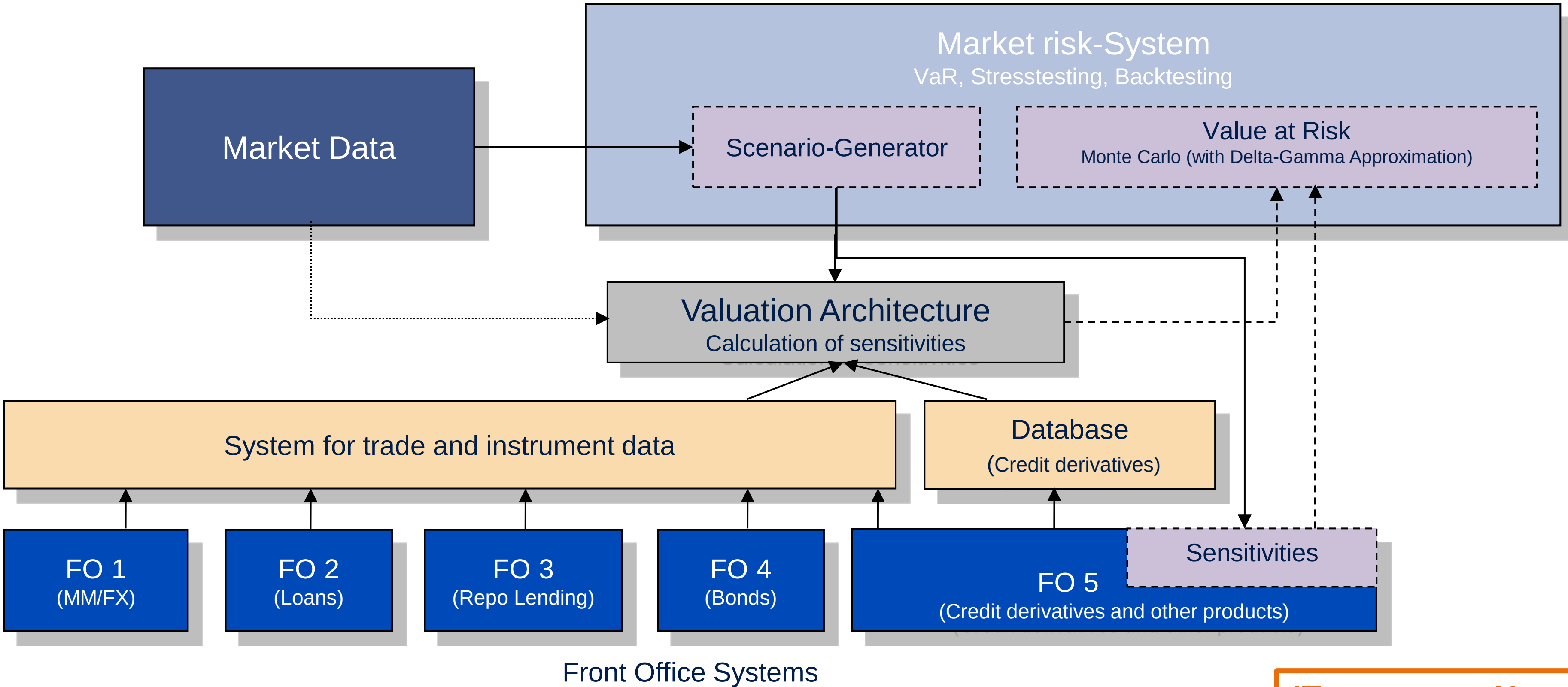


Complexity of Calculations

- » For large portfolios, a huge number of trade valuations is required for VaR calculation
- » Example historical simulation of a large bank: 100 Million trade valuations per day !
- » Even larger number for MC VaR: typically 2000-10,000 MC scenarios (instead of 250 for HistVaR)
- » Securities should be valuated by position, not by trade, but some trading systems can not handle positions.
- » Effort required for efficient processes and implementation ! (e.g. Quasi-MC-methods)



Complexity of a Market Risk Architecture



IT systems ~ N
Interfaces ~ N^2

Trade, market and instrument data from all systems have to be brought into the same form and fed into the market risk engine.

Complexity of Errors – the quest of reasoning...

» Every day banks are required to calculate and report their market risk to their board. So everyday business demands that some risk controllers look at VaR-numbers calculated during the night. Their main job is to analyse numbers, which do not look plausible.

The Hunting Ground of Errors – there are many possible sources

- Wrong trade data
- Wrong market data
- Wrong address data
- Wrong instrument data
- IT Error in a system

- Error in an interface
- Error of valuation model
- Error of risk model
- Error of manual process

- Number is correct, but position changed a lot
- **... and sometimes even no errors at all**
- Number is correct, but market data changed a lot

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